

# A Full-Order Sliding Mode Flux Observer with Stator and Rotor Resistance Adaptation for Induction Motor

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**Abstract**—For resolving the imprecise flux estimation caused by parameters variation of induction motor, a full-order sliding mode flux observer with stator and rotor resistance adaptation is proposed in this paper by constructing the state equations with the stator currents and rotor flux as variables. This scheme identifies stator and rotor resistances during flux estimation, updating the changing stator and rotor resistance of flux estimation system to ensure the robustness of the whole parameter estimation operation. Taking the advantage of the second order low-pass filter characteristic of the designed observer, the high-frequency switching noise contained in the flux can be filtered without the addition of extra low-pass filter. The observer is applied to the direct torque control (DTC) of induction motor to achieve good control performance. The simulation results validate the feasibility and effectiveness of the proposed algorithm.

**Keywords**—induction motor drive; full-order sliding mode observer; flux observer; online resistance estimation

## I. INTRODUCTION

Induction motor (IM) has been widely used in electric traction fields for the advantages of good stability, simple structure, low cost and wide speed range [1]. Common control methods that have been introduced for induction motor are Field Oriented Control (FOC) and Direct Torque Control (DTC). Compared to the FOC, DTC has attracted extensive attention due to its fast dynamic response and simple control structure [2].

The DTC takes the torque and the stator flux as the controlled variables, and estimates the torque by stator currents and stator flux. Therefore, the control performance of DTC system heavily depends on accurate knowledge of the stator flux. However, stator flux is generally assigned as unmeasured internal state. In traditional DTC system, the stator flux is estimated by integrating the back electromotive force. Unfortunately, the use of a pure integrator will cause the drift problem due to the dc offset in measured currents. On the other hand, the stator resistance variation also deteriorates the performance of the integrator estimator, especially when the motor operates at low speed [3]. Therefore, in order to improve the flux observer performance, in recent years, the scholars have proposed many flux observers, such as the improved voltage model observer [3,4], Luenberger observer [5,6], Extended Kalman Filter (EKF) observer [7], Neural Network

Observer [8]. Nevertheless, these methods are sensitive to the changes of motor parameters, which reduces the accuracy of flux estimation.

Compared to the ordinary control methods, sliding mode control has attractive advantages of robustness to external disturbances and low sensitivity to the system parameter variations [9-14]. However, the chattering phenomenon exists in the conventional sliding mode control. The usual method is using a low-pass filter to smooth the current signals for flux estimation. Unfortunately, it introduces the phase lag and the amplitude attenuation at the same time. In [10], a high-order sliding mode observer was proposed to estimate rotor flux for induction motor and eliminate the chattering phenomena. Nevertheless, it has been noted that the algorithm is limited due to the real-time complicated implementation. In addition, sliding mode flux observer is susceptible to the rotor resistance variation, but few works have dealt with this problem. In [15], an MRAS structure for rotor resistance adaptation was proposed. However, the method assumes that the stator resistance remains constant, which does not match the actual motor operating conditions.

In order to address these problems, in this paper, an adaptive full-order sliding mode flux observer with online adjustment of rotor and stator resistance is proposed. The flux observer is quite robust to the variations of stator and rotor resistance. Meanwhile, the high-frequency switching noise contained in the flux can be filtered without the addition of extra low-pass filter, and the phase lag caused by the low-pass filter can be avoided. Furthermore, the adaptation laws are introduced into the observer, which are derived based on Lyapunov function. The adaptive observer benefits from computational simplicity and can be implemented easily in industrial induction motor drive. The simulation results demonstrate the validity and effectiveness of the proposed algorithm.

## II. ADAPTIVE FULL-ORDER SLIDING MODE FLUX OBSERVER DESIGN FOR IM

### A. DTC of IM and Machine Equations

The block diagram of adaptive full-order sliding mode flux observer-based direct torque control system of IM is shown as Fig. 1.

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$$V_1 = 0.5 \left( \sigma L_s (\tilde{i}_{sa}^2 + \tilde{i}_{sb}^2) + \frac{\tilde{\eta}^2}{k_r} + \frac{\tilde{R}_s^2}{k_s} \right) \quad (8)$$

where  $k_r > 0$ ,  $k_s > 0$  are design parameters.

Differentiating  $V_1$  with respect to time, it gets

$$\dot{V}_1 = \sigma L_s (\tilde{i}_{sa} \dot{\tilde{i}}_{sa} + \tilde{i}_{sb} \dot{\tilde{i}}_{sb}) + \tilde{\eta} \dot{\tilde{\eta}} / k_r + \tilde{R}_s \dot{\tilde{R}}_s / k_s \quad (9)$$

From (5) and (7), then

$$\begin{aligned} \dot{V}_1 = & -\tilde{i}_{sa} (m_1 \text{sign}(\tilde{i}_{sa}) - L_m \eta \tilde{\psi}_{ra} / L_r - L_m \omega_r \tilde{\psi}_{rb} / L_r) \\ & -\tilde{i}_{sb} (m_2 \text{sign}(\tilde{i}_{sb}) - L_m \eta \tilde{\psi}_{rb} / L_r + L_m \omega_r \tilde{\psi}_{ra} / L_r) \\ & + \tilde{\eta} (\dot{\tilde{\eta}} / k_r + L_m (\tilde{i}_{sa} \tilde{\psi}_{ra} + \tilde{i}_{sb} \tilde{\psi}_{rb} - L_m i_{sa} \tilde{i}_{sa} - L_m i_{sb} \tilde{i}_{sb}) / L_r) \\ & + \tilde{R}_s (\dot{\tilde{R}}_s / k_s - i_{sa} \tilde{i}_{sa} - i_{sb} \tilde{i}_{sb}) - k_1 \tilde{i}_{sa}^2 - k_2 \tilde{i}_{sb}^2. \end{aligned}$$

If the estimated error of  $\eta$  and stator resistance are chosen to be as follows:

$$\begin{cases} \dot{\tilde{\eta}} = -\frac{k_r L_m}{L_r} (\tilde{i}_{sa} \tilde{\psi}_{ra} + \tilde{i}_{sb} \tilde{\psi}_{rb} - L_m i_{sa} \tilde{i}_{sa} - L_m i_{sb} \tilde{i}_{sb}) \\ \dot{\tilde{R}}_s = k_s (i_{sa} \tilde{i}_{sa} + i_{sb} \tilde{i}_{sb}) \end{cases} \quad (10)$$

then, the terms in brackets are zero.

Consequently, if the observer gains  $k_1$ ,  $k_2$ ,  $m_1$  and  $m_2$  are chosen large enough such that

$$k_1 > 0, \quad k_2 > 0, \quad m_1 > \max \left\{ \left| \frac{L_m}{L_r} \eta \tilde{\psi}_{ra} \right| + \left| \frac{L_m}{L_r} \omega_r \tilde{\psi}_{rb} \right| \right\},$$

$$m_2 > \max \left\{ \left| \frac{L_m}{L_r} \eta \tilde{\psi}_{rb} \right| + \left| \frac{L_m}{L_r} \omega_r \tilde{\psi}_{ra} \right| \right\},$$

then we have  $\dot{V}_1 < 0$ , which guarantees that the current estimation errors converge to zero. This completes the proof.

According to the equations (6) and (10), the estimated rotor resistance and the estimated stator resistance can be expressed as

$$\begin{cases} \dot{\hat{R}}_r = L_r \dot{\tilde{\eta}} = -\frac{k_r L_m}{L_r} (\tilde{i}_{sa} \tilde{\psi}_{ra} + \tilde{i}_{sb} \tilde{\psi}_{rb} - L_m i_{sa} \tilde{i}_{sa} - L_m i_{sb} \tilde{i}_{sb}) \\ \dot{\hat{R}}_s = k_s (i_{sa} \tilde{i}_{sa} + i_{sb} \tilde{i}_{sb}) \end{cases} \quad (11)$$

The adaptation mechanism in (11) can be adjusted by PI regulator.

$$\begin{cases} \hat{R}_r = (k_{p1} + \frac{k_{i1}}{s}) (-\tilde{i}_{sa} \tilde{\psi}_{ra} - \tilde{i}_{sb} \tilde{\psi}_{rb} + L_m i_{sa} \tilde{i}_{sa} + L_m i_{sb} \tilde{i}_{sb}) + R_r(0) \\ \hat{R}_s = (k_{p2} + \frac{k_{i2}}{s}) (i_{sa} \tilde{i}_{sa} + i_{sb} \tilde{i}_{sb}) + R_s(0) \end{cases} \quad (12)$$

where  $k_{p1}$ ,  $k_{p2}$ ,  $k_{i1}$  and  $k_{i2}$  are proportional and integral regulator (P-I) gain.

Once the estimation errors converge to zero, the current estimation error equations in (7) can be expressed as

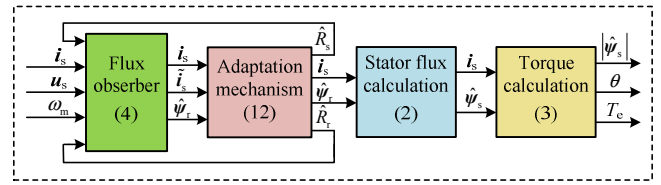


Fig. 2. The block diagram of adaptive flux and torque observer

$$\begin{cases} 0 = \frac{L_m}{L_r} \eta \tilde{\psi}_{ra} + \frac{L_m}{L_r} \omega_r \tilde{\psi}_{rb} - m_1 \text{sign}(\tilde{i}_{sa}) \\ 0 = \frac{L_m}{L_r} \eta \tilde{\psi}_{rb} - \frac{L_m}{L_r} \omega_r \tilde{\psi}_{ra} - m_2 \text{sign}(\tilde{i}_{sb}) \end{cases} \quad (13)$$

In order to ensure the estimation error  $\tilde{\psi}_{ra}$  and  $\tilde{\psi}_{rb}$  can converge to zero, the parameters  $k_3$ ,  $k_4$ ,  $m_3$  and  $m_4$  of observer should be selected reasonably.

*Proof:* The following Lyapunov function candidate is considered:

$$V_2 = 0.5 (\tilde{\psi}_{ra}^2 + \tilde{\psi}_{rb}^2). \quad (14)$$

Differentiating  $V_2$  with respect to time, it gets

$$\begin{aligned} \dot{V}_2 = & \tilde{\psi}_{ra} \dot{\tilde{\psi}}_{ra} + \tilde{\psi}_{rb} \dot{\tilde{\psi}}_{rb} \\ = & -\frac{L_m}{L_r} \left( \frac{m_3}{m_1} \eta \tilde{\psi}_{ra}^2 + \frac{m_3}{m_1} \omega_r \tilde{\psi}_{ra} \tilde{\psi}_{rb} + \frac{m_4}{m_2} \eta \tilde{\psi}_{rb}^2 - \frac{m_4}{m_2} \omega_r \tilde{\psi}_{ra} \tilde{\psi}_{rb} \right) \\ & - \eta (\tilde{\psi}_{ra}^2 + \tilde{\psi}_{rb}^2). \end{aligned} \quad (15)$$

Consequently, if  $k_3$ ,  $k_4$ ,  $m_3$  and  $m_4$  are chosen as:  $k_3 > 0$ ,  $k_4 > 0$ ,  $m_3/m_1 = m_4/m_2 > 0$ , then we have  $\dot{V}_2 < 0$ , which guarantees that the flux estimation errors converge to zero. This completes the proof.

The block diagram of proposed adaptive full-order sliding mode flux observer is shown as Fig. 2.

### III. CHATTERING SUPPRESSION ANALYSIS

Sliding mode observer is well known for the robustness to external disturbances and low sensitivity to the system parameters variation. However, the chattering phenomenon will lead to low estimation accuracy and excite high frequency dynamics. The usual method is using a low-pass filter to smooth the current signals for flux estimation. Unfortunately, it introduces the phase lag and the amplitude attenuation at the same time. The full-order sliding mode observer presented in this paper can solve this problem properly.

According to the flux observer in (4), the following equations can be obtained:

$$\begin{cases} \hat{\psi}_{ra} = \frac{1}{s^2 + \eta s + \omega_r^2} \left( \frac{s L_m}{T_r} \hat{i}_{sa} - \frac{L_m}{T_r} \omega_r \hat{i}_{sb} + \eta \omega_r \hat{\psi}_{rb} + s g_1 + \omega_r g_2 \right) \\ \hat{\psi}_{rb} = \frac{1}{s^2 + \eta s + \omega_r^2} \left( \frac{L_m}{T_r} \omega_r \hat{i}_{sa} + \frac{s L_m}{T_r} \hat{i}_{sb} - \eta \omega_r \hat{\psi}_{ra} + \omega_r g_1 + s g_2 \right) \end{cases} \quad (16)$$

where  $s$  denotes the Laplace operator.

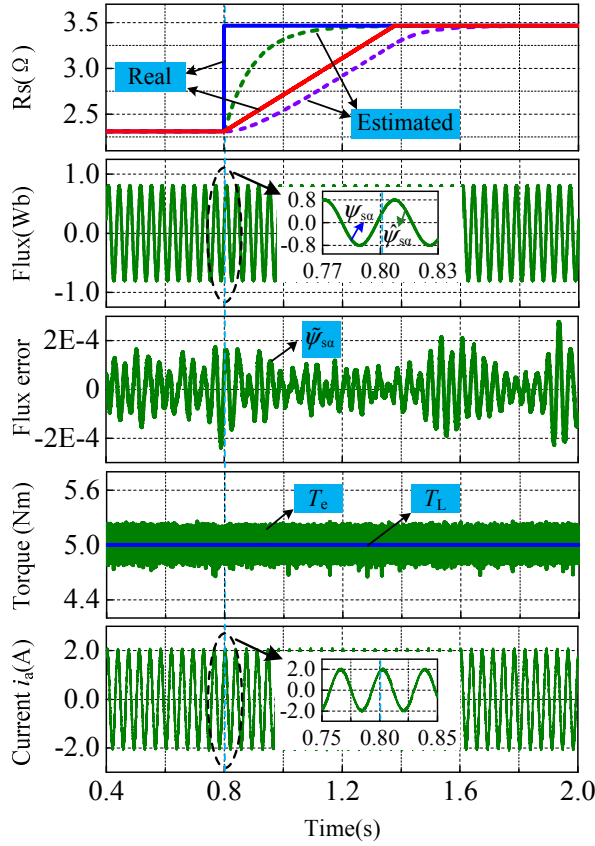


Fig. 3. The simulation results for stator resistance change with stator resistance estimation

From (16), it can be clearly seen that  $1/(s^2 + \eta s + \omega_c^2)$  represents a second order low-pass filter as an inherent part of the designed observer. Therefore, the high-frequency switching noise contained in the flux can be filtered without the addition of extra low-pass filter.

#### IV. SIMULATION RESULTS

In order to validate the effectiveness of proposed adaptive full-order sliding mode flux observer, the corresponding simulation model is conducted by using MATLAB/Simulink. The DC-link voltage  $U_{dc}$  is 480V, the torque load  $T_L$  is 5Nm, the speed of motor is 800r/min, and the main motor parameters are listed in Table I.

TABLE I. MAIN PARAMETERS OF MOTOR

Parameters	Values
Rated power	2.2kW
Rated voltage	380V
Rated current	5.2A
Rated torque	15Nm
Stator resistance	2.31Ω
Rotor resistance	1.94Ω
Mutual inductance	273.92mH
Pole pairs	2

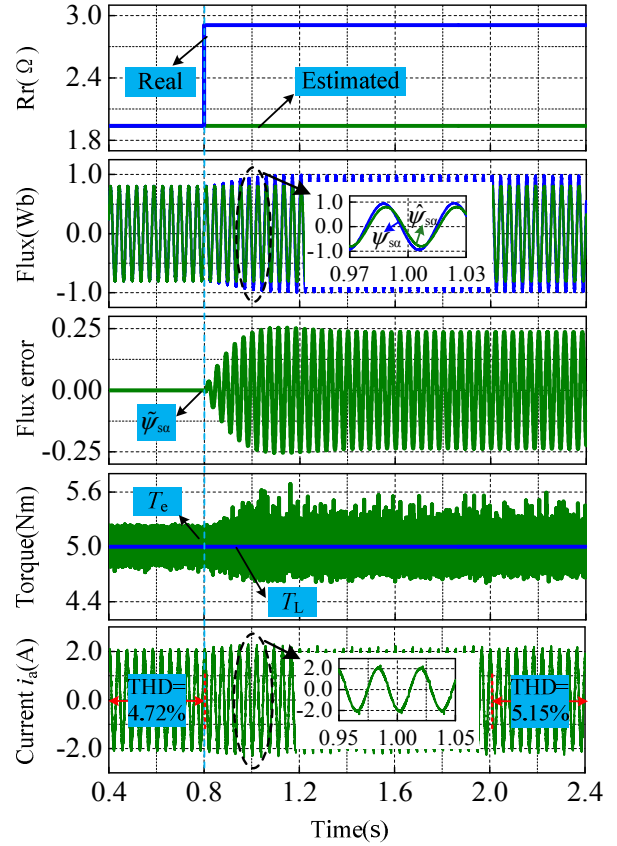


Fig. 4. The simulation results for rotor resistance change without rotor resistance estimation

##### A. Experiment I

As shown in Fig. 3, at  $t=0.8s$ , the stator resistance was increased by 50% in nominal value with a step change and ramp change, respectively.

It can be seen in Fig. 3 and Fig. 7a that the estimator can accurately estimate the stator resistance in a short time with the designed adaptation mechanism. Meanwhile, the change of stator resistance does not have any influence on the estimated value of stator flux, the torque and the output current. The simulation results demonstrate the proposed observer has a better robustness to the stator resistance variation.

##### B. Experiment II

Fig. 4 shows the changes of stator flux observation, the output torque and output current when the rotor resistance was increased by 50% in nominal value with a step change. Fig. 7b shows the corresponding stator flux trajectory.

It can be seen in Fig. 4 that there is an obvious deviation between the estimated value and actual value of stator flux. In addition, the output current becomes unbalanced, current harmonics and torque ripple increase significantly when the rotor resistance changes suddenly. From the Fig. 7b, it is easy to find that the amplitude of stator flux is larger than the given value, which may lead to an adverse effect on the operation of the motor.

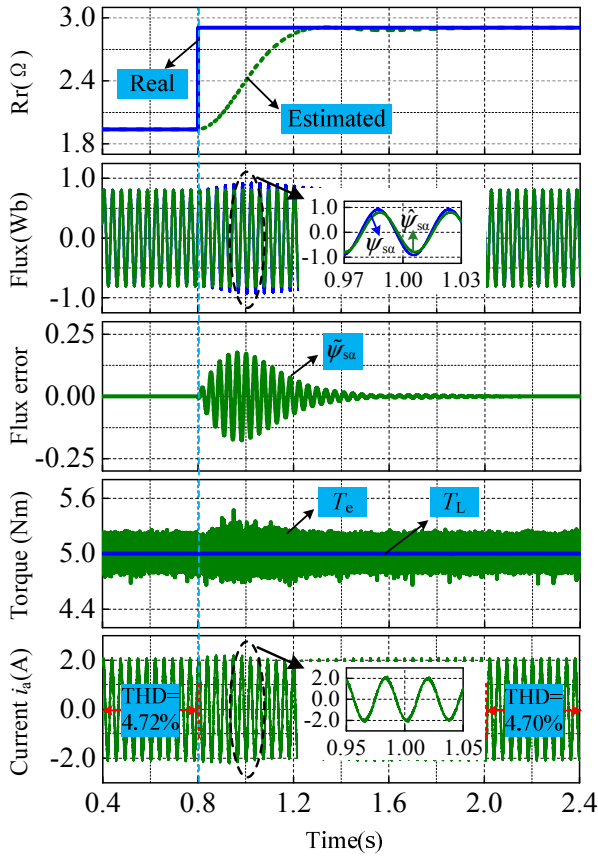


Fig. 5. The simulation results for a step change of rotor resistance with rotor resistance estimation

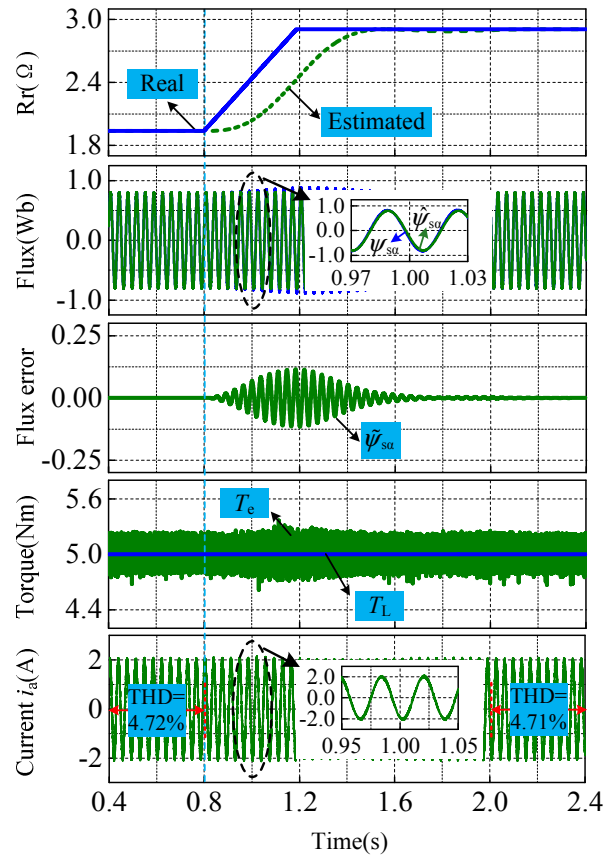


Fig. 6. The simulation results for a ramp change of rotor resistance with rotor resistance estimation

As shown in Fig. 5 and Fig. 6, at  $t=0.8s$ , the rotor resistance is increased by 50% in nominal value with a step change and ramp change, respectively. However, the observer is improved by online rotor resistance estimation.

It can be seen in Fig. 5 that the estimator can accurately estimate the rotor resistance in a short time with the designed adaptation mechanism. During the estimation of resistance, there is a small deviation between the estimated value and actual value of stator flux, and the deviation can eventually converge to zero. In addition, the current harmonics and torque ripple have a slightly increased when the rotor resistance changes suddenly, but the current and torque can return to its original performance when the estimated rotor resistance converges the actual resistance. Fig. 7c displays the change of trajectory of stator flux and finally the flux trajectory is back to the initial trajectory.

Compared to Fig. 5 and Fig. 7c, Fig. 6 and Fig. 7d show that estimation errors of stator flux, the current harmonics, torque ripple and the amplitude of stator flux have a smaller increment, which means the proposed adaptive flux observer with online adjustment of rotor resistance has a better performance when the resistance changes slowly. The simulation results demonstrate the proposed observer has a better robustness to the rotor resistance variation.

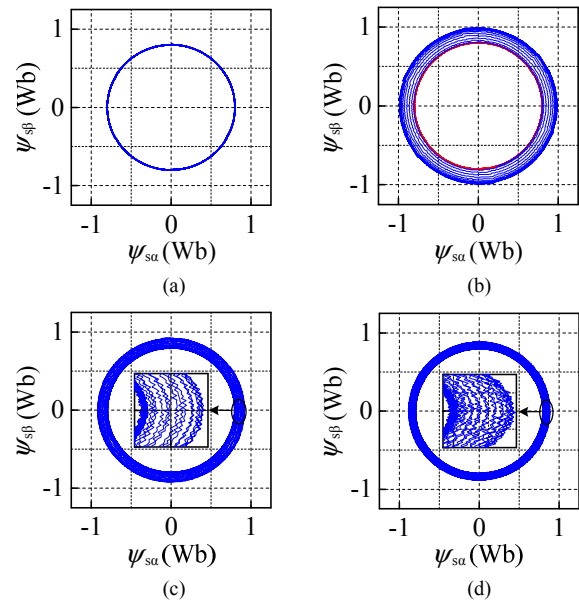


Fig. 7. The stator flux trajectory of (a) stator resistance change, (b) a step change of rotor resistance without rotor resistance estimation, (c) a step change of rotor resistance with rotor resistance estimation and (d) a ramp change of rotor resistance with rotor resistance estimation.

## V. CONCLUSIONS

This paper has proposed a full-order sliding mode flux observer with stator and rotor resistance online adaptation which can be derived based on Lyapunov function. It is shown that the estimated stator resistance and rotor resistance can converge to the corresponding actual values. Meanwhile, the observer is able to suppress the chattering without the addition of extra low-pass filter, and the phase lag caused by the low-pass filter can be avoided. The simulation results show the feasibility and effectiveness of the proposed method.

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