

A Novel Adaptive Control for Three-phase Inverter

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Abstract—This paper proposes a simple and effective adaptive voltage control for the three-phase inverter with LC filter. In the proposed adaptive control, the control input is artificially defined as two types: *power control input* and *signal control input*. A state feedback, load current feedforward and reference feedforward are imposed to the *power control input* to improve the dynamic performance. A dynamic adaptive gain of the reference feedforward is proposed as the *signal control input* to reject the parameter uncertainty and disturbance. The proposed control method establishes excellent voltage regulation such as fast transient response, zero steady-state error, and low total harmonic distortion. The simulation and experimental results verify the good performance and robustness of the proposed adaptive control.

Keywords—Inverter control; adaptive control; complex signal;

I. INTRODUCTION

Output voltage control is critical for inverters in distributed energy resources (DERs) [1], [2], or in uninterruptible power supply systems [3]. A lot of efforts have been dedicated to the inverter control, and many control algorithms have been proposed.

The techniques for the control of inverters can be classified into two types: linear and nonlinear methods. For the linear methods, the tracking controllers, for example the proportional plus integrator or resonant (PI/R) controller, are often adopted. Most early control algorithms are designed in frequency-domain based on small signal transfer function. Those control methods are either multi-loop control [3]-[6] or single-loop control [7]-[8] and are widely employed today. Lately, time-domain based optimal control methods are applied to the inverter voltage control [9]-[14]. Basically, different optimization objectives, such as maximum convergence rate [9], minimum H_∞ norm [10], [11], and minimum linear quadratic index regulation (LQR) [12]-[14], are constructed to design the controller gains. Those approaches achieve excellent performance at the expense of a more deliberate parameter design.

Advanced nonlinear control techniques are also used to control the voltage of the inverter. A model predictive control technique is applied for uninterruptible-power-supply (UPS) applications [15] [16]. An extended Lyapunov-function-based

control is proposed to ensure the global stability of the closed-loop system in [17]. While in [18]-[20], some adaptive voltage control methods are proposed to accomplish good performance.

In those approaches [15]-[20], the internal model principle based controller (PI/R controller) is usually absent, hence the control law should be designed carefully to achieve zero-steady-state-error control. Moreover, for those nonlinear approaches [15]-[20], the stability is always evaluated via Lyapunov method. However it is not easy to utilize Lyapunov method, because there are no systematic approaches to find an effective Lyapunov function. Particularly, the adaptive control are generally highly nonlinear as proposed in [18]-[20], more mathematical knowledge is needed to design the adaptive control law.

So far, for the inverter control, the previous tracking feedback controls all utilize the middle point voltage of the inverter leg as the control input, namely the *power control input*. It is natural and normal. Differently, this paper proposes a simple adaptive control where a concept of *signal control input* is proposed for the tracking feedback control instead of the conventional *power control input*. The *signal control input* is actually a real-time digital signal processed in the digital signal processor (DSP). This processing allows the proposed adaptive control to be a linear system, and its control law and stability can be tuned and analyzed via linear system knowledge. Moreover, since only a simple state feedback is imposed on the *power control input*, the proposed method also achieves good dynamic response, high stability and robustness. The simulation and experimental results verify the validity of the proposal.

II. PROPOSED ADAPTIVE CONTROL

A. Complex Model for Thress-Phase Inverters

To design the voltage control, the model of the inverter with an LC filter is studied first. Fig. 1 shows the typical topology of a three-phase inverter. The state space model of the plant under α - β or dq reference frame is given by

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_1\mathbf{v}_c + \mathbf{B}_2\mathbf{w} \\ \mathbf{y} = \mathbf{C}\mathbf{x} \end{cases} \quad (1)$$

In this model, the complex state variable \mathbf{x} : $[\mathbf{i}_{La\beta} \ \mathbf{u}_{Ca\beta}]^T = [i_{La} + j i_{L\beta} \ u_{Ca} + j u_{C\beta}]^T$ or $[\mathbf{i}_{Ldq} \ \mathbf{u}_{Cdq}]^T = [i_{Ld} + j i_{Lq} \ u_{Cd} + j u_{Cq}]^T$ denotes the inductor current and capacitor voltage, and \mathbf{w} : $\mathbf{i}_{La\beta} = i_{La} + j i_{L\beta}$ or $\mathbf{i}_{Ldq} = i_{Ld} + j i_{Lq}$ represents the disturbance from the load current. Furthermore the voltage at the middle point of the leg \mathbf{v}_c : $\mathbf{v}_{ca\beta} = v_{ca} + j v_{c\beta}$ or $\mathbf{v}_{cdq} = v_{cd} + j v_{cq}$ is defined as the **power control input** to distinguish the later **signal control input**. The relationship between the variables under α - β and dq reference is: $\mathbf{v}_{dq} = \mathbf{v}_{a\beta} e^{-j\omega t}$, where ω is the fundamental frequency. Defining $a_1 = 1/L$ and $a_2 = 1/C$, the matrices in the model can be easily derived:

$$\mathbf{A}: \mathbf{A}_{a\beta} = \begin{bmatrix} -Ra_1 & -a_1 \\ a_2 & \end{bmatrix} \text{ or } \mathbf{A}_{dq} = \begin{bmatrix} -Ra_1 - j\omega & -a_1 \\ a_2 & -j\omega \end{bmatrix}, \quad (2)$$

$$\mathbf{B}_1 = \begin{bmatrix} a_1 \\ 0 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} 0 \\ -a_2 \end{bmatrix}, \mathbf{C} = [0 \ 1]$$

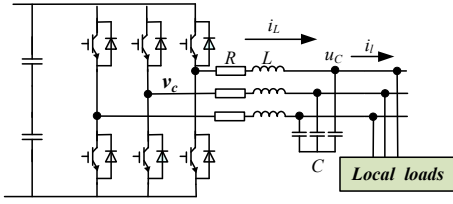


Fig. 1. Topology of the three-phase inverter.

It is worth mentioning that the adoption of the complex signals is important for the control. It enables the whole three-phase system to be considered as integrated unit to design the complex gains. The detailed illustration will be shown in the next section.

B. Adaptive Control

The proposed adaptive control strategy is shown in Fig. 2. The state feedback law, which is the unique feedback imposed on the **power control input**, is used to improve the dynamic response of the system. The disturbance feedforward \mathbf{k}_d can be used to shape the output impedance. The complex gain \mathbf{k}_r is adopted to ensure unity gain of the transfer function from reference to output at frequency of 50 or 0 Hz for α - β or dq reference frame control. The key idea of this paper is to design $\mathbf{k}_r = \mathbf{k}_{r0} + \mathbf{k}_{ra}$ where \mathbf{k}_{ra} is a dynamic adaptive complex gain referred to the **signal control input** (since \mathbf{k}_{ra} is considered as a real-time signal here) to compensate the parameter uncertainty and load variation and also eliminate the steady state error.

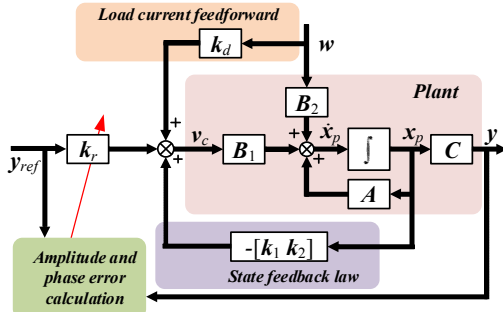


Fig. 2. Proposed adaptive control.

Ignoring the dynamic adaptive gain first, the **power control input** in α - β reference frame is $\mathbf{v}_c = -\mathbf{K}\mathbf{x} + \mathbf{k}_d \mathbf{w} + \mathbf{k}_{r0} \mathbf{y}_{ref}$, where $\mathbf{K} = [\mathbf{k}_1 \ \mathbf{k}_2]$. Then substituting \mathbf{v}_c into (1), the closed-loop transfer function is calculated as

$$\begin{aligned} \mathbf{y} &= T_{ra\beta} \mathbf{y}_{ref} + T_{wa\beta} \mathbf{w} \\ &= \mathbf{C} (\mathbf{sI} - \mathbf{A}_{a\beta} + \mathbf{B}_1 \mathbf{K})^{-1} (\mathbf{B}_1 \mathbf{k}_{r0} \mathbf{y}_{ref} + (\mathbf{B}_2 + \mathbf{B}_1 \mathbf{k}_d) \mathbf{w}_p) \\ &= \frac{a_1 a_2 \mathbf{k}_{r0} \mathbf{y}_{ref} + [a_1 a_2 \mathbf{k}_d - a_2 (\mathbf{s} + a_1 R + a_1 \mathbf{k}_1)] \mathbf{w}}{s^2 + a_1 (R + \mathbf{k}_1) s + a_2 a_1 (1 + \mathbf{k}_2)} \end{aligned} \quad (3)$$

To make sure that $T_{ra\beta}$ has unity gain at fundamental frequency, the coefficient \mathbf{k}_{r0} can be determined by

$$T_{ra\beta}(s) \Big|_{s=j\omega} = \frac{a_1 a_2 \mathbf{k}_{r0}}{s^2 + a_1 (R + \mathbf{k}_1) s + a_2 a_1 (1 + \mathbf{k}_2)} \Big|_{s=j\omega} = 1 \quad (4)$$

Correspondingly, \mathbf{k}_d can be tuned by shaping $T_{wa\beta}$ to zero at any desired frequency:

$$T_{wa\beta}(s) \Big|_{s=jn\omega} = \frac{a_1 a_2 \mathbf{k}_d - a_2 (\mathbf{s} + a_1 R + a_1 \mathbf{k}_1)}{s^2 + a_1 (R + \mathbf{k}_1) s + a_2 a_1 (1 + \mathbf{k}_2)} \Big|_{s=jn\omega} = 0 \quad (5)$$

where n can be any integer. The physical meanings of (4) and (5) are distinct.

Observing the state transition matrixes of $\mathbf{A}_{a\beta}$ and \mathbf{A}_{dq} , the transfer function under dq reference frame can be acquired as: $T_{rdq}(s) = T_{ra\beta}(s + j\omega)$, $T_{wdq}(s) = T_{wa\beta}(s + j\omega)$. Apparently, $T_{rdq}(s) \Big|_{s=0} = T_{ra\beta}(s) \Big|_{s=j\omega}$, it means that \mathbf{k}_{r0} derived from (4) also makes T_{rdq} own unity gain at 0 Hz. This is important to make the two controls equivalent.

In practice, \mathbf{k}_{r0} derived from (4) may not ensure $T_{ra\beta}(s) \Big|_{s=j\omega}$ exactly equals one due to the parameter uncertainties and load variations. To reject those uncertainties, the dynamic adaptive gain \mathbf{k}_{ra} is proposed to response the amplitude and phase error. Rewriting \mathbf{k}_r in polar and rectangular formations respectively:

$$\begin{aligned} \mathbf{k}_r &= A_r e^{j\theta_r} = \mathbf{k}_{r0} + \mathbf{k}_{ra} = (A_{r0} + A_{ra}) e^{j(\theta_{r0} + \theta_{ra})} \quad .a \\ \mathbf{k}_r &= k_{rRe} + j k_{rIm} = \mathbf{k}_{r0} + \mathbf{k}_{ra} = k_{r0Re} + j k_{r0Im} + k_{raRe} + j k_{raIm} \quad .b \end{aligned} \quad (6)$$

Basically, the magnitude and phase angle of T_{rdq} or $T_{ra\beta}$ are independently related to A_r and θ_r respectively. Therefore \mathbf{k}_{ra} , referred to **signal control input**, can be adjusted to eliminate the steady-state-error:

$$A_{ra} = k_m \int (E_{ref} - E_C) dt, \theta_{ra} = k_{ph} \int (\varphi_{ref} - \varphi_C) dt \quad (7)$$

where E_{ref} and φ_{ref} are the magnitude and phase of the reference: $\mathbf{y}_{ref} = E_{ref} e^{j\varphi_{ref}}$, and accordingly $\mathbf{y} = \mathbf{u}_C = E_C e^{j\varphi_C}$ is the output voltage. However, the computation of (6).a is nonlinear if the adaptive law (7) is adopted. It is undesirable for tuning the adaptive gains k_m and k_{ph} . Considering that θ_{r0} derived from (4) is very small, namely, $k_{r0Re} \gg k_{r0Im}$, and critically, \mathbf{k}_{ra} has a powerful ability to regulate the output voltage (k_{raRe} and k_{raIm} are always very small). Then it has $A_{ra} \approx k_{raRe}$ and $\theta_{ra} \approx k_{raIm}$. Consequently, the following adaptive law can be utilized:

$$k_{raRe} = k_m \int (E_{ref} - E_C) dt, k_{raIm} = k_{ph} \int (\varphi_{ref} - \varphi_C) dt \quad (8)$$

The advantage of this adaptive law is to make the signal operation linear and easier, because (6).b is linear.

So far, another important issue to be solved is the acquisition of the magnitude and phase error. Considering the Park transformation operator $e^{j\varphi_{ref}}$, it has $\mathbf{u}_{Cdq} = E_C e^{j\varphi_C} e^{-j\varphi_{ref}} = E_C e^{j(\varphi_C - \varphi_{ref})}$. Obviously, u_{cd} represents the magnitude of the voltage, and u_{cq} inherently includes the phase error. Ultimately, the adaptive law evolves to

$$k_{raRe} = k_{apt} \int (E_{ref} - u_{cd}) dt, k_{raIm} = k_{apt} \int -u_{cq} dt \quad (9)$$

The practical realization is shown in Fig. 3. This formation of the adaptive law removes the nonlinear link from the tracking feedback loop. Then the design of the scalar gain k_{apt} becomes easy.

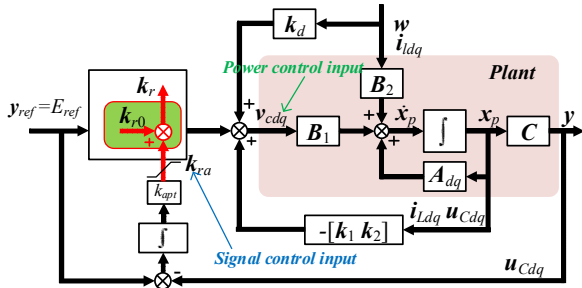


Fig. 3. Practical realization of the adaptive control.

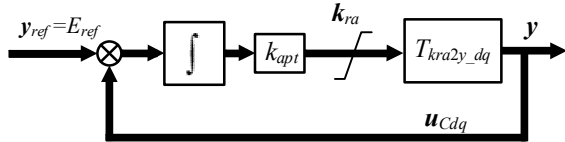


Fig. 4. Equivalent block diagram for k_{apt} design.

III. VERIFICATION

To verify the proposed control algorithm, the simulation and experiment are performed with a 5 kW load. Before the verification, the parameter design and control implementation are illustrated.

A. Parameter Design

To sum up, there are four complex and one scalar coefficients need to be determined: k_{r0} , k_d , k_1 , k_2 and k_{apt} . Among those coefficients, only the state feedback k_1 and k_2 will affect the stability of the system when the limitation is imposed on k_{ra} , because only this feedback is imposed on the **power control input**. This is an attractive feature, the stability can be easily evaluated.

The calculation of k_{r0} and k_d has been illustrated previously as (4) and (5). The state feedback law k_1 and k_2 can be tuned by the well-known LQR which can be easily fulfilled by the function `lqr()` supplied by MATLAB. It is worth noting that the LQR will lead to the same state feedback law for $A_{\alpha\beta}$ and A_{dq} . The critical task is to tune k_{apt} . When designing k_{apt} , k_{ra} is a time-variant signal as **signal control input**. In this condition,

the transfer function from k_{ra} to the output can be easily acquired:

$$T_{kra2y_dq} = \frac{C(s\mathbf{I} - \mathbf{A}_{dq} + \mathbf{B}_1\mathbf{K})^{-1} \mathbf{B}_1 E_{ref}}{a_1 a_2 E_{ref}} \quad (10)$$

$$= \frac{1}{(s + j\omega)^2 + a_1(R + \mathbf{k}_1)(s + j\omega) + a_2 a_1(1 + \mathbf{k}_2)}$$

which is actually the control objective of the **signal control input** as shown in Fig. 4. Therefore the equivalent block diagram shown in Fig. 4 can be used to tune the adaptive gain.

For the simulation and experiment in this paper, the nominal filter parameters used are: $L=1\text{mH}$, $C=60\mu\text{F}$. The control parameters are: $k_1=6.9364$, $k_2=0.4142$, $k_d=6.9864-j1.571$, $k_{r0}=1.4083+j0.1317$. Gain k_{apt} is set to 3 based on Bode plot analysis of the open loop shown in Fig. 4. As shown in Fig. 5, a 73.2 degree of phase margin is acquired with $k_{apt} = 3$.

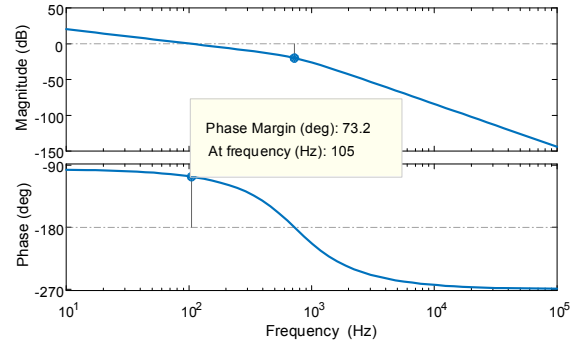


Fig. 5. Bode diagram for k_{apt} design.

B. Implementation

Since the complex signals are adopted in the controller, the operation of complex number needs to perform:

$$k_{ra} = k_{apt} \int (E_{ref} - \mathbf{u}_{Cdq}) dt \quad (11)$$

$$\mathbf{v}_{cdq} = -\mathbf{k}_1 \mathbf{i}_{Ldq} - \mathbf{k}_2 \mathbf{u}_{Cdq} + \mathbf{k}_d \mathbf{i}_{dq} + (k_{r0} + k_{ra}) \mathbf{y}_{ref}$$

Hence, for the proposed controller, two integrators (used to generate k_{ra}), four complex multiplications and four complex additions need to be realized as described by (11). In the simulation, an s-function based on C-code is adopted to fulfill the controller. For the experiment, a TMS320F28335-based control board is used to measure the voltage and current signals and realize the proposed controller.

C. Results

The simulation and experimental results for the nominal parameters are shown in Fig. 6. A 5 kW load is used to test the performance of the control method. As shown in Fig. 6, the simulation and experimental results demonstrate that the proposed adaptive control works very well. When the load changes, the adaptive law modifies the **signal control input** to reject the load variation. Although k_{raRe} and k_{raIm} are very small, the steady-state-error is eliminated and the transient response is very fast. It proves that the **signal control input** has a powerful control ability.

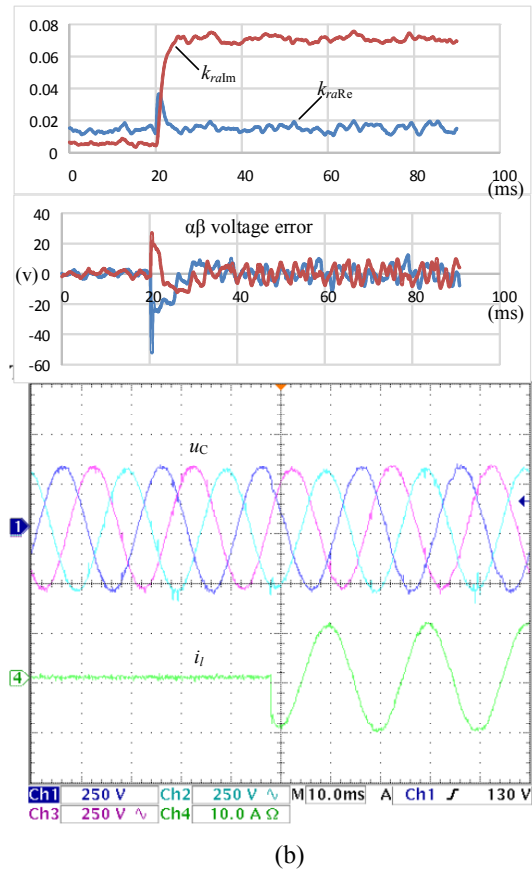
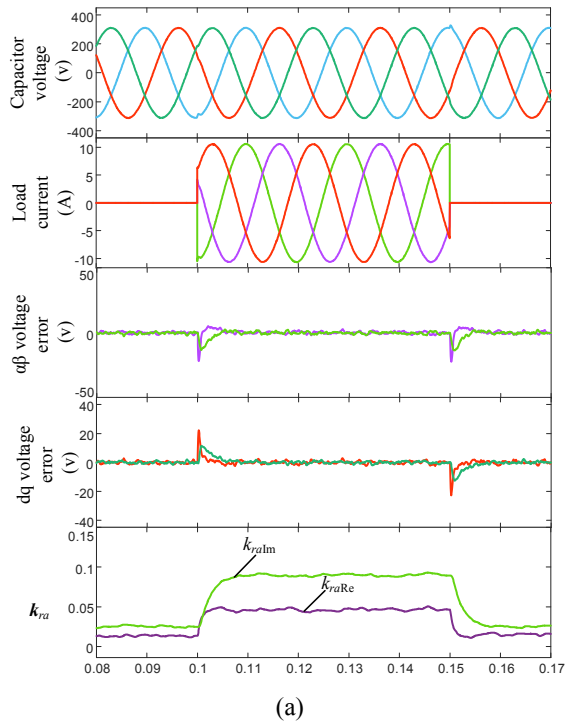


Fig. 6. Results of the proposed adaptive control under normal parameters ($L=1\text{mH}$, $C=60\mu\text{F}$): (a) simulation results; (b) experiment results.

Moreover, the simulation for parameter variations is performed to verify the rejection ability to the uncertainties. The result for the parameters of $L=2\text{mH}$, $C=30\mu\text{F}$ is shown in Fig. 7. In this parameters, the gain of T_{rdq} at 0 Hz is different from the nominal parameters, the value of k_r to guarantee the unity gain in 0 Hz is also different. However, with the proposed adaptive mechanism, the integrators can find the ultimate value of k_r to guarantee the unity gain in 0 Hz regardless the initial gain of T_{rdq} . This is why the proposed adaptive control has strong robustness. As shown in Fig. 7, the value of k_{ra} generated by the integrator is different from the one in nominal condition. But the performance is still good, because with the proposed adaptive control, the inverter is actually changed to a high gain link (the control objective of the signal control input), consequently, although the value of k_{ra} is very small, it can adjust the output voltage effectively.

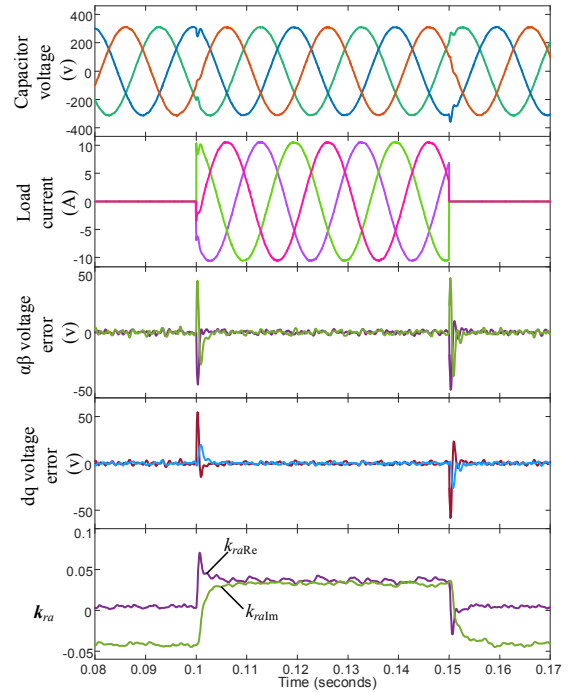


Fig. 7. Simulation results of the proposed adaptive control under parameter variations: $L=2\text{mH}$, $C=30\mu\text{F}$.

IV. CONCLUSION

The concept of *power control input* and *signal control input* are proposed in this paper. Some effective controls such as state feedback, load current and reference feedforward can be imposed onto the *power control input* to improve the dynamic performance of the system. While the *signal control input* is used to eliminate the steady-state-error and improve the robustness. The *signal control input* is benefit for the stability and robustness thanks to its virtual isolation function (isolating the tracking control output from the *power control input*). The proposal also manifests a good performance. The effectiveness and feasibility is verified by simulations and experiments. Moreover, the introduction of the *signal control input* offers a different thinking to design classical PI/R controllers for the inverter control.

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