Simplified Discrete-Time Modeling and Dynamic Characteristics Analysis of PI-Controlled Voltage Source Inverter

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***Abstract*—The characteristic of voltage source inverter under PI-control is much more complex than under P-control. When system parameters are not properly designed, this system may appear unstable oscillation at different frequency. The stability boundary of control parameters have both upper and lower limit. In previous works on discrete-time modeling and analysis, P-controller is always taken as object of study for simplicity, which is different from practical applications where PI-controller is often adopted. Therefore, previous works have limited research significance. In this paper, a simplified discrete-time model of voltage source inverter under PI-control is derived. Based on analysis of the proposed model, different unstable phenomenon and mechanism corresponding to upper and lower stability boundary have been revealed. Moreover, high accuracy stability boundary has been derived and verified by experiment.**

Keywords—discrete-time model; bifurcation; digital control; state-space average

I. INTRODUCTION

Discrete-time model has been applied in modeling and analysis of digital controlled switching power converters to study both their slow- and fast-scale dynamic characteristics [1]-[6]. Due to the additionally introduced state-variable and subsequent complexity of integrator, most of the previous works on discrete-time modeling and analysis only pay attention to the characteristic of power stage itself [7]-[10]. Therefore, P-controller is always taken as object of study to simplify the analysis. Obviously, PI-controller is much more often adopted in practical applications.

There is huge difference between the characteristic of PI-controlled system and P-controlled system, owing to the difference in system order. Therefore, previous works on discrete-time modeling and research of systems under P-control have limited research significance.

Simplified discrete-time model of PI-controlled voltage source inverter is derived in this paper. Based on analysis of the model, mechanism of unstable oscillation at different frequency have been revealed. This paper is organized as follows. Section II is system description and working principle. Section III is simplified discrete-time modeling and analysis. Section IV gives experimental results. Section V presents the conclusions of this work.

II. SYSTEM DESCRIPTION AND WORKING PRINCIPLE

The structure of voltage source inverter is shown in Fig. 1. The load voltage v_c is controlled to track its reference v_{ref}, which is sinusoidal. PI-controller and P-controller are adopted for voltage loop and current loop, respectively. Detailed block diagram of digital controller is shown in Fig. 2.

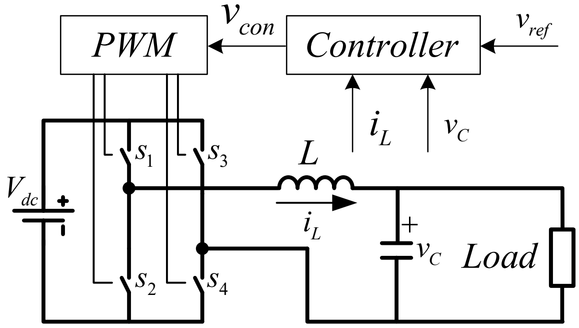

Fig. 1. Voltage source inverter.

Supported by the National Natural Science Foundation of China (51707160), and the Fundamental Research Funds for the Central Universities (3102016OQD0114).

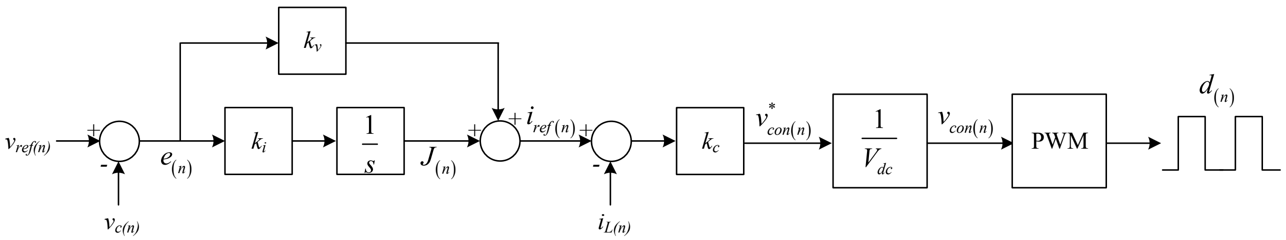

Fig. 2. Block diagram of digital controller.

In Fig.2, J denotes the state of the integrator in PI-controller, v^*_{con} is the output of P-controller in current loop, v_{con} is the normalized input for Pulse-Width-Modulation. Subscript n indicates that variable is sampled at the beginning of n-th switching cycle. Digital symmetric PWM strategy is employed, whose modulation strategy is shown in Fig. 3. System parameters are listed in TABLE I.

III. SIMPLIFIED DISCRETE-TIME MODELING AND ANALYSIS

According to Fig. 1 and Fig. 3, state equation of power stage can be derived.

$$\begin{cases} \dfrac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}_1 V_{dc} \\ \dfrac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}_2 V_{dc} \end{cases} \tag{1}$$

where,

$$\mathbf{A} = \begin{bmatrix} 0 & -\dfrac{1}{L} \\ \dfrac{1}{C} & -\dfrac{1}{RC} \end{bmatrix}; \quad \mathbf{B}_1 = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix};$$

$$\mathbf{B}_2 = \begin{bmatrix} -\dfrac{1}{L} \\ 0 \end{bmatrix}; \quad \mathbf{B}_{\mathrm{avg}(n)} = \begin{bmatrix} \dfrac{2d_{(n)} - 1}{L} \\ 0 \end{bmatrix}$$

Using simplified discrete-time modeling method [8][9], simplified discrete-time model of power stage is derived.

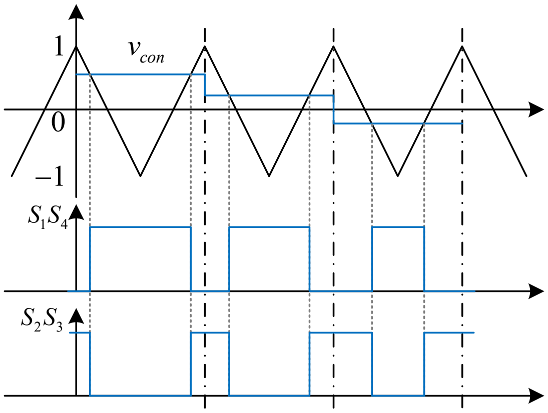

Fig. 3. Digital symmetric PWM strategy.

TABLE I. SYSTEM PARAMETERS

V_{dc}	L	C	R	f_s
200 V	3.5 mH	20 uF	2.6 ohm	10 kHz

$$\mathbf{x}_{(n+1)} = e^{\mathbf{A}T_s}\mathbf{x}_{(n)} + (e^{\mathbf{A}T_s} - \mathbf{I})\mathbf{A}^{-1}\mathbf{B}_{\mathrm{avg}(n)}V_{dc} \tag{2}$$

According to the block diagram in Fig. 2 and Forward Euler discretization method, state variable of the integrator in PI-controller and the duty cycle can be expressed as:

$$J_{(n)} = e_{(n-1)} \times k_i \times T_s + J_{(n-1)} \tag{3}$$

$$d_{(n)} = k_c\left[k_v e_{(n)} + J_{(n)} - i_{L(n)}\right]\frac{1}{2V_{dc}} + \frac{1}{2} \tag{4}$$

Combine equation (2), (3) and (4), simplified discrete-time model of the whole system is obtained.

$$\begin{cases} \mathbf{x}_{(n+1)} = e^{\mathbf{A}T_s}\mathbf{x}_{(n)} + (e^{\mathbf{A}T_s} - \mathbf{I})\mathbf{A}^{-1}\mathbf{B}_{\mathrm{avg}(n)}V_{dc} \\ J_{(n+1)} = J_{(n)} + k_i\left(i_{ref(n)} - i_{L(n)}\right)T_s \end{cases} \tag{5}$$

The corresponding 3rd order Jacobian matrix is shown below.

$$Jacobian = \begin{bmatrix} J_1 + k_c m & J_2 + k_c k_v m & -k_c m \\ J_4 + k_c n & J_5 + k_c k_v n & -k_c n \\ 0 & -k_i T_s & 1 \end{bmatrix} \tag{6}$$

where,

$$\begin{bmatrix} J_1 & J_2 \\ J_4 & J_5 \end{bmatrix} = e^{AT_s}; \quad m = J_2 + \frac{J_1 - 1}{R}; \quad n = J_5 - 1 + \frac{J_4}{R}$$

Relationship between the magnitude of eigenvalues and control parameters are depicted in Fig. 4. Obviously, for particular k_i and k_c, system will be unstable not only when k_v is too large, but also when it is too small.

Trajectory of eigenvalues is shown in Fig. 5. When k_v is too small, conjugated eigenvalues fall outside unit circle, indicating Hopf bifurcation. With the increase of k_v, conjugated eigenvalues move into unit circle. As k_v continues to increase, conjugated eigenvalues go through unit circle at different points, predicting Hopf oscillation at different frequency [2][11]. The above analysis proves and explains unstable oscillation at different frequency in this system.

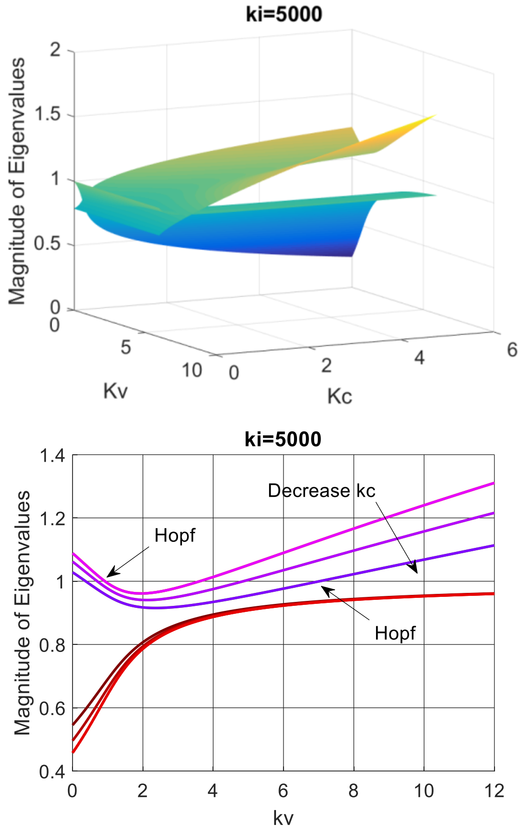

Fig. 4. Magnitude of eigenvalues versus control parameters.

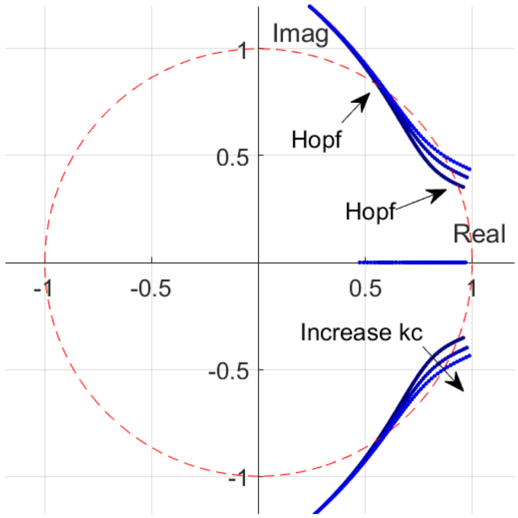

Fig. 5. Eigenvalue trajectory.

Stability boundary of control parameters can be easily obtained from Fig. 4. Take k_i and k_c equal to 5000 and 1.4 as example, the critical value of k_v corresponding to Hopf bifurcation at different frequency are 0.7 and 4.86, respectively.

IV. EXPERIMENTAL VERIFICATION

Experimental waveform of load voltage and inductor current corresponding to different k_v are shown in Fig. 6. When k_v is smaller than 0.7, low frequency Hopf oscillation is observed. When k_v is between 0.7 and 4.86, system is stable; When k_v is larger than 4.86, high frequency Hopf oscillation is observed. Theoretical results are verified by experiment.

V. CONCLUSIONS

This paper derived a simplified discrete-time model of PI-controlled voltage source inverter. Phenomenon and mechanism of unstable oscillation at different frequency have been revealed. Stability boundary of control parameter is also obtained. The results of theoretical analysis have been experimentally verified.

Previous works on discrete-time modeling only pay attention to the characteristic of power stage itself for simplicity. P-controller is always taken as object of study. Using simplified discrete-time model, this paper accurately studied the behavior of a voltage source inverter under PI-control. Since PI-controller is much more often adopted in practical applications, this work has greater research significance.

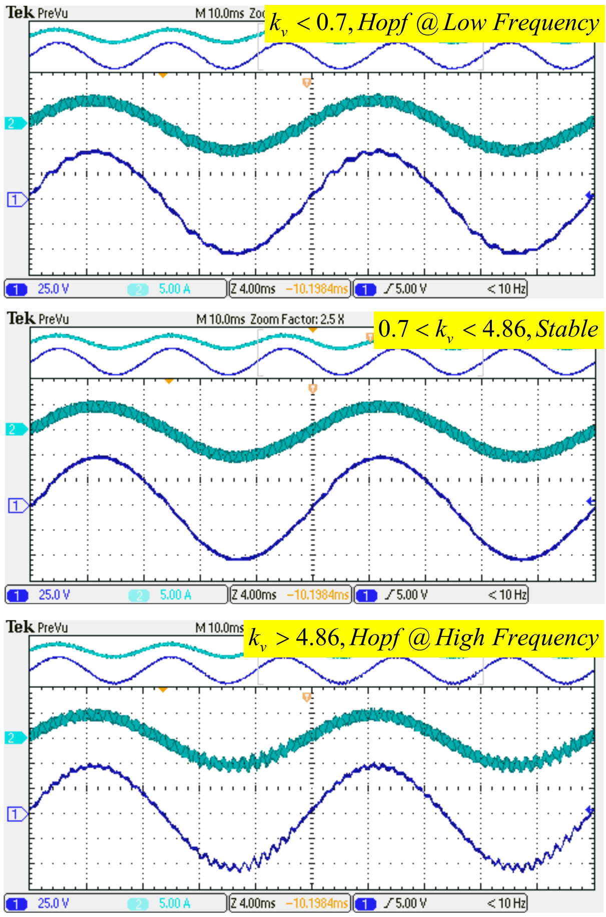

Fig. 6. Experimental waveforms.

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