Discrete Time Adaptive State Feedback Control of DC-DC Power Electronic Converters

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***Abstract*—DC-DC power electronic converters are widely used in modern electrical systems due to their broad range of applications in smart grids, electric vehicles and drive systems. A discrete time, adaptive state feedback controller for a Buck DC-DC power converter operates in the continuous conduction mode is proposed in this paper. Considering a resistive output load, discrete time dynamical model of the average mode buck converter is obtained. Then, a discrete time state feedback adaptive algorithm is developed to regulate the output voltage of an uncertain converter without exact knowledge of the converter output load, input voltage and converter parameters. The adaptive law is derived based on the normalized gradient decent algorithm to adaptively update the feedback gains. Simulation study and the corresponding experimental test cases were carried out to analyze the performance and the effectiveness of the developed controller.**

I. Introduction

With the developments in smart power grids, electric drive systems and electric vehicles, application of power electronic converters (PECs) in modern electrical and industrial systems has become extremely popular [1]–[5]. These PECs require controllers to obtain the desired output according to the application. Most popular and widely utilized controller is the PI controller. Even though, it is the most simple and easiest algorithm, it suffers from poor transient performances [6] and it requires information of all the system parameters. Hence, advance algorithms are required to control PECs with guaranteed stability and under unknown system parameters.

Various types of control methodologies have been proposed during past decades to facilitate control actions in PECs based on nonlinear control architecture, predictive control, optimal and game theoretic control [7]–[11]. Assuming the exponential form of the linear multiloop controller, a nonlinear controller is proposed in [7] to regulate the output voltage of boost and buck converters by providing an additional tuning parameter to modify the output response. Considering the precise dynamical model of the buck converter including the parasitic resistances of the inductor, capacitor and switches, a design of a closed loop controller is presented in [8] based on the exact feedback linearization. Application of optimal control to regulate the voltage and power in PECs can be found in [9]. Here PECs are modeled as distributed generators operating in a DC microgrid. Game theoretic controllers are becoming popular in control systems domain and its application in PECs are found in [10] and [11]. Optimal state trajectories and control of boost PECs are obtained in [10] based on the Pontryagin's minimum principle while in [11] an online feedback strategy is proposed. Further, assuming the current information of the converter is unavailable, a reduced order observer based output feedback controller is designed in [12] to control DC–DC buck PEC.

Most of the above mentioned methods require all the system parameters and state information to develop the controller. However, when there are uncertainties in the system dynamics or when the system dynamics are unavailable, adaptive type controllers are required to facilitate the control aspect of the PECs [13]. Application of adaptive techniques in PECs can be found in the literature [14]–[17]. Motivated by the passive nature of the DC-DC power converters, passivity based adaptive controllers are designed for the boost and buck-boost converters in [14]. Continuous time average mode dynamical system operates in continuous conduction mode is considered when deriving the system dynamics. The output load resistance is assumed unknown and output voltage regulators are developed considering full state information structure. To tackle the unknown load and input voltage issues in power converters, an adaptive pulse width modulation (PWM) strategy has been proposed in [15] based on the sliding mode control (SMC) to regulate the output voltage of a boost converter. The adaptive control law in the proposed feedback control methodology consists of an observer based state estimator. An adaptive feed forward compensation technique is proposed in [16] for the boost power converter to address the uncertainty issues in the system dynamics and input voltage. It utilizes a two degree of freedom feedback topology while preserving the robustness to attain proper output voltage control under system disturbances. Intelligent control mechanisms are also available in power converter adaptive control arena. In [17], an adaptive fuzzy neural control methodology is developed for the voltage tracking of the boost power converter. In order to improve the robustness of the converter during the transient stage, a total sliding mode control approach is utilized.

Under the DC-DC PECs, one of the most popular converter in solar PV and DC microgrid area is the buck converter. Improved adaptive and robust controller designed for the buck converter can be found in the literature [18]–[20]. In [18] a neural network based adaptive decentralized controller for the buck converter is proposed which operates as the

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interface between PV array and a microgrid. Here the dynamic model is converted to the feedback linearizable form and the neural network universal approximation property [21] is used to approximate the unknown dynamics of the system. An adaptive backstepping controller is designed for the buck converter in [19] where, a Chebyshev neural network [22] based technique is developed to regulate the output voltage under to compensate the uncertainties of the loads. Further, an adaptive, finite time controller for voltage regulation in buck converter is proposed in [20]. In this approach, both the input voltage and output load are assumed to be unknown. In order to estimate the unknown parameters in a finite time, two finite-time convergent observers are designed in the controller.

Most of the previous work related to adaptive control of PECs require an offline training or exact knowledge of the input voltage and converter parameters. Further, the discrete time feedback adaptive controllers for PECs are limited in the literature. Therefore, this paper proposes a discrete time adaptive controller to regulate the output voltage of the buck converter without exact knowledge of the system parameters. The proposed adaptive algorithm in this paper does not require any offline training, full information of input voltage, output load and parameters of the converter. It takes the system states as input and provide control actions based on the estimated feedback gains in discrete domain.

The rest of the paper is organized as follows. In section II, mathematical modeling of the buck power converter and the proposed controller is derived. Simulation and experimental verifications are given in sections III and IV respectively. Finally the paper is concluded in section V.

II. Dynamic Modeling of the Converter and the Controller

Mathematical modeling of the buck power converter and the proposed adaptive controller is given in this section. The schematic model of the buck converter can be shown as in Fig. 1. The two states in the system are inductor current I_L and capacitor voltage V_c and the control input is the duty cycle D. The output voltage of the converter is equal to the capacitor voltage. The parameters L, C, V_{in} are the inductance of the inductor, capacitance of the capacitor, and input voltage of the converter.

Define the states $x_1 = V_c$, $x_2 = I_L$ and the control $u = D$, the state space representation of the converter can be derived as [23],

$$\dot{x} = Ax + Bu \ ; \quad y = \bar{C}x \tag{1}$$

where, $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$, $A = \begin{bmatrix} -1/RC & 1/C \\ -1/L & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ V_{in}/L \end{bmatrix}$ and $\bar{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}$. This continuous time state space model can be converted to discrete time state space model as shown in (2) [24].

$$x(k+1) = Gx(k) + Hu(k) \ ; \quad y(k) = \bar{C}x(k) \tag{2}$$

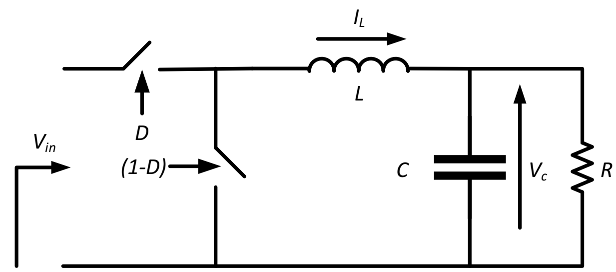

Fig. 1. Schematic diagram of the DC-DC buck converter.

Here, $G = e^{AT}$ and $H = (e^{AT} - I_{2\times 2})A^{-1}B$. T is the sampling time, k is the discrete time step. System matrix (A) is invertible since determinant of A is non zero. The input output relationship of the open loop system in complex z domain can be shown as [24], [25],

$$y(z) = F(z)U(z) \tag{3}$$

where, z is the complex variable and $F(z)$ is the pulse transfer function which can be expressed as [25],

$$F(z) = C(zI - G)^{-1}H = k_p \frac{Z(z)}{P(z)} \tag{4}$$

The numerator and denominator polynomials of the pulse transfer function $Z(z)$ and $P(z)$ are second and first order functions respectively. Hence the system relative degree $n^\star = 1$. Suppose the desired output voltage is generated by the following system.

$$y_m(k) = W_m(z)\gamma(k) \tag{5}$$

Here, $\gamma(k)$ is the external reference input signal and $W_m(z)$ is a stable transfer function with degree $n^\star$ given by,

$$W_m(z) = \frac{1}{P_m(z)} = \frac{1}{z^{n^\star}} \tag{6}$$

The proposed adaptive controller takes the form as in (7).

$$u^\star(k) = (K_1^\star)^T x(k) + K_2^\star \gamma(k) \tag{7}$$

Where K_1^* and K_2^* are the ideal weights of the feedback gains. With this feedback, the close loop system of (2) can be written as,

$$x(k+1) = (G + H(K_1^*)^T)x(k) + HK_2^*\gamma(k) \tag{8}$$

Then, the output of the system can be written as,

$$Y(z) = M(z)\Upsilon(z) \tag{9}$$

$$M(z) = \bar{C}(zI - G - H(K_1^\star)^T)^{-1}HK_2^\star = \frac{Z(z)K_2^\star}{det(zI - G - H(K_1^\star)^T)} \tag{10}$$

where, $\Upsilon(z)$ is the z transform of $\gamma(k)$ and I is the identity matrix with appropriate dimension.

If we select the gains $K_1^\star$ and $K_2^\star$ such that, $det(zI - G - H(K_1^\star)^T) = P_m(z)Z(z)/k_p$ and $K_2^\star = 1/k_p$, then as far as $Z(z)$ is stable, the term $P_m(z)Z(z)$ is stable. Therefore, the output can be shown as,

$$Y(z) = W_m(z)\Upsilon(z) \ \rightarrow \ y(k) = \gamma(k - n^\star) \tag{11}$$

However, to compute actual gains, we need the complete information of the system. Since the system dynamics are unknown, an estimate of these gains K_1 and K_2 are used to obtain the feedback control signal shown in (12).

$$u(k) = K_1^T(k)x(k) + K_2(k)\gamma(k) \tag{12}$$

With this estimated feedback, the closed loop dynamics can be shown as,

$$\begin{aligned} x(k+1) = (G + H(K_1^\star)^T)x(k) + HK_2^\star\gamma(k) + H[(K_1^T(k) \\ - (K_1^\star)^T)x(k) + (K_2(k) - K_2^\star)\gamma(k)] \end{aligned} \tag{13}$$

Using (9), (10) and (13), output signal can be written as [25],

$$\begin{aligned} &y(k) = \bar{C}(G + H(K_1^\star)^T)^k x(0) + W_m(z)\gamma(k) \\ &+ \frac{W_m(z)}{K_2^\star}[(K_1^T(k) - (K_1^\star)^T)x(k) + (K_2(k) - K_2^\star)\gamma(k)] \end{aligned} \tag{14}$$

Here, the term $\bar{C}(G + H(K_1^\star)^T)^k x(0)$ exponentially converges to zero due to the stability of $G + H(K_1^\star)^T$ according to [25]. Therefore, using (5) and (14) the error can be written as,

$$\begin{aligned} e(k) = y(k) - y_m(k) = \rho^\star W_m(z)[(K_1^T(k) - (K_1^\star)^T)x(k) \\ + (K_2(k) - K_2^\star)\gamma(k)] \end{aligned} \tag{15}$$

where, $\rho^\star = \frac{1}{K_2^\star} = k_p$ and define its estimation as $\rho(k)$. Taking the unknown gain vector to be estimated as $\theta(k) = [K_1^T(k) \quad K_2(k)]^T$, the estimation error $\varepsilon(k)$ can be written as,

$$\varepsilon(k) = e(k) + (\rho(k) - \rho^\star)\xi(k) \tag{16}$$

where, $\xi(k) = \theta(k)^T\zeta(k) - W_m(z)\theta(k)^T\omega(k)$, $\zeta(k) = W_m(z)\omega(k)$ and $\omega(k) = [x(k) \ \gamma(k)]^T$. Utilizing (15), the error $\varepsilon(k)$ can be rewritten as,

$$\varepsilon(k) = \rho^\star(\theta(k) - \theta^\star)^T\zeta(k) + (\rho(k) - \rho^\star)\xi(k) \tag{17}$$

where, $\theta^\star = [(K_1^\star)^T \ K_2^\star]^T$ is the ideal feedback gain vector. Based on the normalized gradient method, the following adaptive laws are suggested to tune the unknown gains [25].

$$\theta(k+1) = \theta(k) - sign[\rho^\star]\frac{\Gamma\zeta(k)\varepsilon(k)}{m^2(k)} \tag{18}$$

$$\rho(k+1) = \rho(k) - \frac{\eta\xi(k)\varepsilon(k)}{m^2(k)} \tag{19}$$

Here, $m^2(k) = (1 + \zeta^T(k)\zeta(k) + \xi^2(k))$ and the constant tuning gains $0 < \Gamma = \Gamma^T < \frac{2}{max(k_p)}I_{3\times3}$ and $0 < \eta < 2$.

Stability of the proposed adaptive law can be shown by taking the Lyapunov function (20) considering $\tilde{\theta}(k) = \theta(k) - \theta^\star$ and $\tilde{\rho}(k) = \rho(k) - \rho^\star$

$$V(\tilde{\theta}, \tilde{\rho}) = |\rho^\star|(\tilde{\theta}(k))^T\Gamma^{-1}\tilde{\theta}(k) + \eta^{-1}\tilde{\rho}(k)^2 \tag{20}$$

Taking the first difference of (20) along the dynamics, time increment of the Lyapunov function can be upper bounded as (21) for some $\alpha_1 > 0$.

$$\begin{aligned} &V(\tilde{\theta}(k+1), \tilde{\rho}(k+1)) - V(\tilde{\theta}(k), \tilde{\rho}(k)) = \\ &-\left(2 - \frac{|\rho^\star|\zeta^T(k)\Gamma\zeta(k) + \eta\xi^2(k)}{m^2(k)}\right)\frac{\varepsilon^2(k)}{m^2(k)} \leq -\alpha_1\frac{\varepsilon^2(k)}{m^2(k)} \end{aligned} \tag{21}$$

The upper bounded inequality follows from the conditions that, $0 < \Gamma = \Gamma^T < \frac{2}{max(k_p)}I_{3\times3}$ and $0 < \eta < 2$ and $|\rho^\star| = |k_p| \leq k_p^0$ where k_p^0 is the upper bound of k_p. According to (21), it can be inferred that, $\theta(k), \rho(k) \in \mathcal{L}^\infty$ and $\theta(k+1) - \theta(k), \rho(k+1) - \rho(k) \in \mathcal{L}^2$ and $\frac{\varepsilon}{m(k)} \in \mathcal{L}^2 \cap \mathcal{L}^\infty$ [25]. Hence according to (16), $e(k) \rightarrow 0$.

III. Simulation Results

The simulations were carried out considering the buck converter parameters $L = 10\ mH$, $C = 120\ \mu F$, $V_{in} = 30\ V$, $R = 20\ \Omega$, $T = 1\ ms$, $\Gamma = 0.002I_{3\times3}$, and $\eta = 1.5$. Desired output voltage was considered as $\gamma(k) = 15\ V$. Test cases were simulated to observe the output voltage regulation in startup, under a reference voltage change, under a load change and under an input voltage change.

A. Output voltage regulation

Converter output voltage regulation in the startup and under a reference voltage change is given in this section. Initially the reference voltage was fixed at $15\ V$. After $0.2\ s$ it was changed to $25\ V$ to observe the adaptability of the controller against reference voltage variations. Resultant output voltage and inductor current are shown in Fig. 2.

As shown in the Fig. 2, converter takes $0.1\ s$ to reach the desired steady state in the startup. Steady state current can be observed as $0.75\ A$. After the reference voltage change, it takes less than $0.05\ s$ to reach and settle at the new equilibrium. Maximum voltage peak and current peak are $29.6\ V$ and $1.48\ A$ which can be observed just after the point of reference voltage change. New steady state current after the change is $1.25\ A$

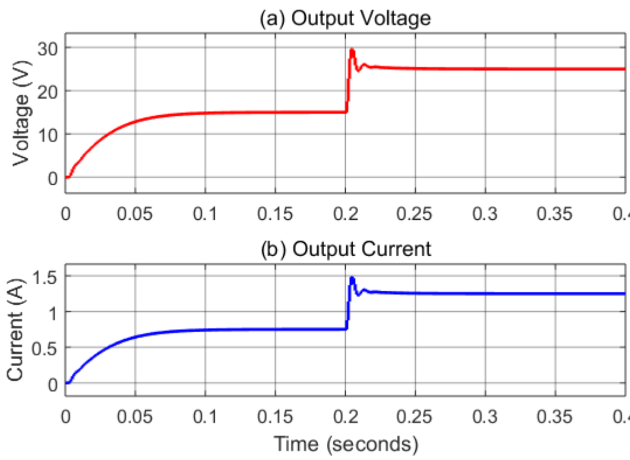

Fig. 2. Output voltage regulation. (a) Output voltage and (b) Output current.

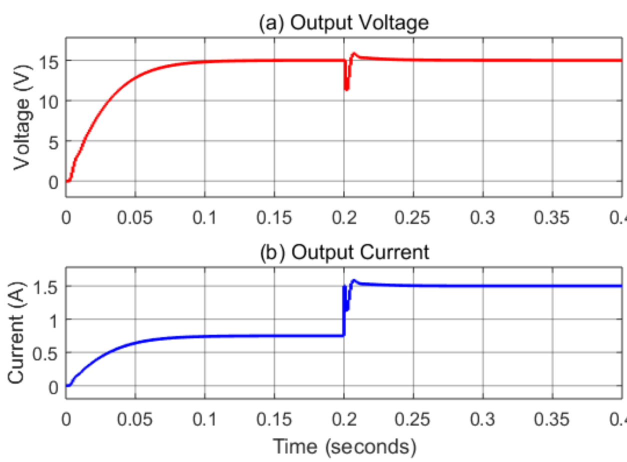

Fig. 3. Output voltage regulation under a load change. (a) Output voltage and (b) Output current.

B. Output voltage regulation under a load change

Adaptability of the proposed controller under a load change is shown in this section. The voltage regulation under a step load change was explored by changing the load from 20 Ω to 10 Ω at $t = 0.2$ s. Variation of output voltage and inductor current are shown in Fig. 3.

According to the given results, output voltage change due to the load disturbance is very small and both the system states reach to their steady states within short period of time. Maximum change in the voltage during the transient is 3.68 V. Steady state inductor current increases to 1.5 A.

C. Output voltage regulation under an input voltage change

Voltage regulation under input voltage disturbances are discussed in this section. To simulate this scenario, a step input voltage change from 30 V to 25 V was given at $t = 0.2$ s and the results are shown in Fig. 4. Both the output current and voltage are affected due to the input voltage disturbance for a short time. Within 0.05 s both current and voltage regain their respective equilibria at 15 V and 0.75 A. Maximum voltage and current dips from the steady state can be observed as 4.28 V and 0.214 A

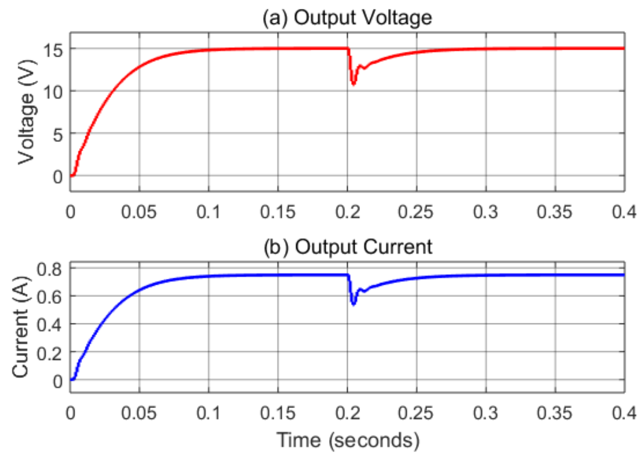

Fig. 4. Output voltage regulation under an input voltage change. (a) Output voltage and (b) Output current.

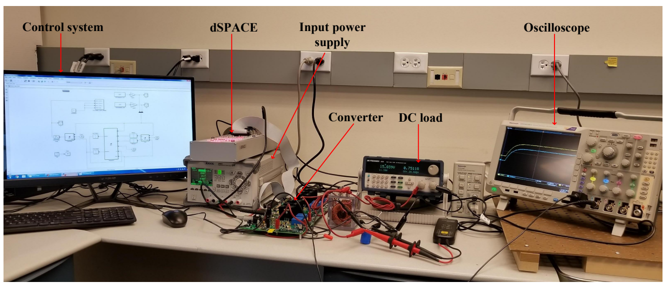

Fig. 5. Experiment test setup.

IV. Experimental Validation

Performance of the proposed controller was experimentally analyzed and results are given in this section. The controller and the converter were implemented based on MATLAB/Simulink and dSPACE control systems. Fig. 5 shows the experimental setup that contains DS1104 controller card, CP1104 I/O board, and MOSFET converter system. Switching frequency of the converter was set to 40 kHz. Corresponding experimental results of the three test cases simulated in the previous section are shown in Fig. 6 to Fig. 8.

Startup transient and reference voltage change experimental result shown in Fig. 6 are closely match with the simulations given in Fig. 2. As shown in the results, output voltage reaches to 15 V and output current reaches to 0.75 A as expected. After the reference voltage change, both output voltage and current gain their respective steady state equilibria within short time of 32 ms. Output voltage and current reach to 25 V and 1.25 A respectively. Maximum voltage peak during the transient can be observed as 29 V and the peak current is 1.3 A.

Experimental output voltage and current variation subjected to a load change is shown in Fig. 7. According to the results,

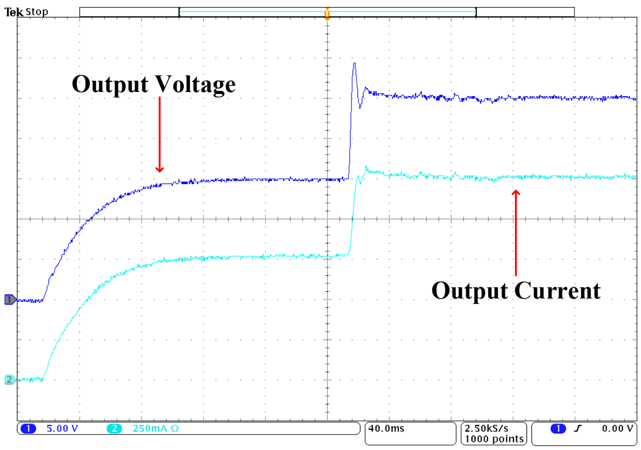

Fig. 6. Output voltage regulation experiment.

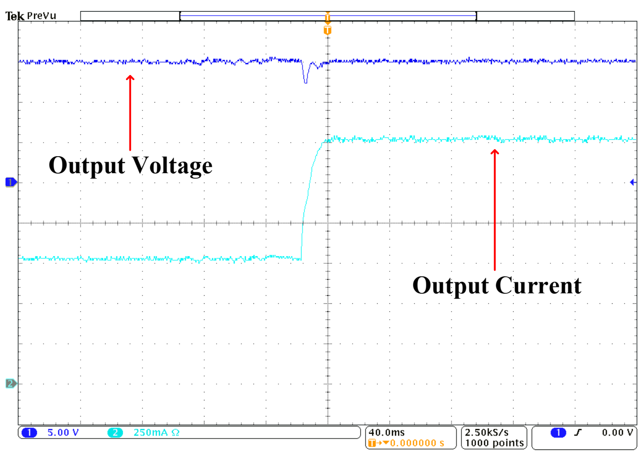

Fig. 7. Output voltage regulation under a load change experiment.

output current changes from $0.75\ A$ to $1.5\ A$ within very short time of $16\ ms$. Corresponding change in the output voltage is small and the maximum voltage dip can be observed as $3\ V$ from the desired value.

Variations in the experimental output current and voltage under an input voltage change is illustrated in Fig. 8. Both the states regain their desired steady states after a very short period of time of $80\ ms$. Maximum deviations of output voltage and current from their respective equilibriums can be seen as $3\ V$ and $150\ mA$.

V. Conclusion

Discrete time adaptive controller for voltage regulation of DC-DC buck power converter has been proposed in this paper. Discrete time dynamical model of the system was obtained by transforming the continuous time state space dynamical system to discrete domain. Then, a discrete time state feedback adaptive controller was designed for the output voltage regulation under unknown system parameters. Unlike most of other adaptive controllers, which require input voltage information, exact knowledge of converter parameters or offline training,

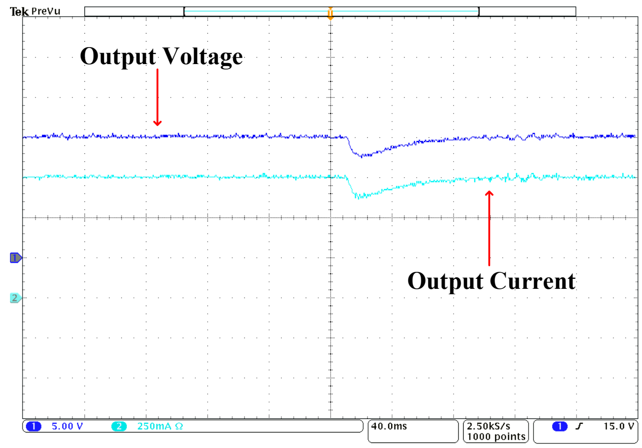

Fig. 8. Output voltage regulation under an input voltage change experiment.

this adaptive scheme only requires the inductor current and capacitor voltage information of the converter. The presented simulation and corresponding experimental results depict the performance of the proposed controller under unknown load, input voltage and other system parameters. Extend the single converter control methodology into coupled multi converter system will be a possible future direction of this work.

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