

# Parameter Estimation of Permanent Magnet Synchronous Machines Based on a New Model Considering Discretization Effects of Digital Controllers

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**Abstract**—Parameter estimation of permanent magnet synchronous motors offers fast adaptation of controllers to changing conditions and thermal monitoring based on tracking resistance estimations. Model-based approaches have been found to be useful toward that goal with a caveat that an accurate estimation can only be achieved with a good model. However, most estimators available in the literature of this kind have been reported to be useful in a restricted operation envelope. In particular, they remain consistent in low-to-moderate speeds and steady-state conditions without compromising accuracy while only a few expands that limited operation envelope with ad-hoc modifications to simplified models, such as capturing divergence of a simplified model from the reality with a look-up table or adding learning terms to a model. In this paper, based on experimental data from both hardware and simulations, we present a limitation of the existing models that might be the central reason to poor performance of parameter estimators during high speed and dynamic motions. Most importantly, we bring an understanding to that limitation and explain the mechanism behind it analytically. Finally, we propose a new model which is more accurate for improved parameter identification.

**Index Terms**—Permanent Magnet Synchronous Motor, Field-Oriented Control, Model-based Parameter Estimation, Least Squares

## I. INTRODUCTION

Permanent magnet synchronous motors (PMSM) are popular in industrial applications due to their high-power density, fast torque-speed response and maintenance free operation. That widespread usage pushes researchers to develop high performance, adaptive, and fault tolerant controllers in order to meet demanding requirements of applications like robotics. In that context, online parameter estimation is an important subject since it enables adaptation of controllers to varying conditions [1] and robustness against thermal and magnetic saturation faults by monitoring electrical parameters of the machine [2].

There are two main approaches to parameter estimation of PMSMs. First body of related work uses estimators that operate in steady-state conditions, hence neglecting voltage

variations due to changes in current [3]. Based on simplified electrical dynamics in those conditions, least-squares observers are used [4]. However, parameters are not completely observable under steady-state conditions [5], causing biased parameter estimation [4] or restricting researchers to estimating only a subset of parameters (e.g., using nominal values of inductances and estimating other parameters) [6], [7]. Specially designed reference signals, for instance, high-frequency d-axis current injection, has been proposed to overcome observability problems with steady-state approaches [5], [8], [9]. The second approach to parameter estimation problem extends capabilities of estimators to transient conditions by exploiting full machine dynamics with a model-based state estimator such as Kalman Filter or Sliding Mode Observer [10] [11] [12].

The paper is organized as follows : In Sec. II-A, based on empirical evidence from both simulation and experiments, we discover a limitation of existing models which deserves a careful consideration in particular for high-speed and dynamic applications (i.e., operating in transient conditions) of PMSMs when used in conjunction with discrete current controllers. In Sec II-B, following a thorough mathematical analysis, we first hypothesize that voltage deviations in the rotating  $dq$  frame during transformation of voltages to stationary  $abc$  frame in a discrete fashion, while the machine rotates, explains the inconsistency between available models and data. Then, we propose a novel model that inherently captures deviations with additional physical phenomenon it includes. In Sec. III, we formulate an optimization problem minimizing the error between predicted currents and measurements, leading to an online parameter estimator based on analytical solutions of motor dynamics which appear nonlinear in terms of motor parameters. In Sec. IV, we conduct systematic simulations in order to assess the performance of that estimator. In particular, estimation accuracy of the new model is evaluated in comparison to an existing model under steady-state conditions, and we assess tracking performance of the estimator assuming that true parameter values vary in time.

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A typical digital field-oriented control (FOC) architecture running at sample time of  $T_s$  is illustrated in Fig. 1. Control actions (i.e., virtual  $dq$  voltages) are first computed in the rotating  $dq$  frame using reference  $dq$  currents and measurements expressed in  $dq$  frame after captured in stationary  $abc$  frame. Then, control actions are expressed in  $abc$  frame, and phase voltages are applied to the motor through a three-phase inverter switched according to a PWM strategy. In that context, assuming balanced conditions (i.e.,  $s_a + s_b + s_c = 0$ ), a quantity  $s$  expressed in  $abc$  frame can be converted to  $dq$  frame, and vice versa via Park's Transformation  $s_{dq} = T_{abc}^{dq}(\theta) s_{abc}$ , which is a function of electrical angle  $\theta$ , with

$$T_{abc}^{dq}(\theta) = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ -\sin(\theta) & -\sin(\theta - 2\pi/3) & -\sin(\theta + 2\pi/3) \end{bmatrix}$$

and inverse Park's transformation  $s_{abc} = T_{dq}^{abc}(\theta) s_{dq}$  with

$$T_{dq}^{abc}(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \cos(\theta - 2\pi/3) & -\sin(\theta - 2\pi/3) \\ \cos(\theta + 2\pi/3) & -\sin(\theta + 2\pi/3) \end{bmatrix} s_{dq}.$$

Model-based parameter estimation of a PMSM integrated with a controller and a driver shown in Fig. 1, as its name implies, needs a good system model for accurate identification. However, existing models ignore nonlinear effects, such as controller dynamics, pulse width modulation effects (PWM), and sensor quantization and noise, thus use an idealized model of the machine. Even though the simplified model has been used often, consequences of simplification in comparison to higher order models has not been measured either qualitatively or quantitatively. In order to bridge this gap, we conducted both hardware and simulation experiments. Our results suggests that prediction accuracy of existing models severely degrades at high speeds in the presence of discrete controllers. In that context, we propose a new model that better captures effects of discrete control actions.

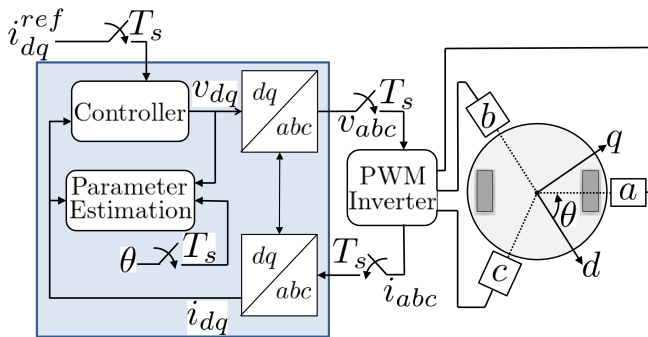


Figure 1: Block diagram of a digital current controller with sampling time  $T_s$  and PMSM with drive electronics.

Electrical dynamics of a non-salient PMSM in  $dq$  coordinates take the form

$$\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} -R/L & \omega \\ \omega & -R/L \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} v_d/L \\ (v_q - \omega \lambda_{pm})/L \end{bmatrix} \quad (1)$$

where  $i$  denotes current,  $v$  voltage,  $L$  inductance,  $R$  resistance,  $\lambda_{pm}$  permanent magnet's flux linkage,  $\omega$  electrical speed i.e.,  $\omega := \dot{\theta}$ , and subscripts  $d$  and  $q$  denote direct and quadrature components of a variable, respectively [13]. Even though Eq. (1) can be directly used for parameter estimation as done in [4], [12], steady-state conditions where  $d(i_{dq})/dt \approx 0$  provides a simplified formulation of dynamics for identification. Note that neither this steady-state model nor the original model in Eq. (1) consider sensor errors that might lead to biased estimate of electrical angle  $\theta$ , effects of PWM inverter, and the fact that voltages are applied discretely, all of which are present in a physical instantiation of a PMSM drive (see Fig. 1).

In references [11], [14], [15], the model's shortcomings are reported as deviations from experimental data in terms of speed dependent prediction errors in high-speed applications and a compensation with a look-up-table or by fitting a function to prediction errors for different velocities and  $dq$  axis currents in the steady-state conditions are proposed. However, understanding of the mechanism causing the discrepancy were left untouched. As a follow-up to those works, we have done steady-state hardware and simulation experiments under no-load conditions with a EC-4pole 30 Maxon Motor controlled by a speed controller with a FOC on a custom-made driver unit. Both speed controller and FOC are PI-type and run at  $5kHz$  and  $10kHz$ , respectively. The PWM frequency is set to  $50kHz$ . Constant speed commands in the range of  $\omega^{ref} \in (0, 3000)$  are applied, and output of FOC corresponding to  $d$  and  $q$  axis voltages are recorded after the system reaches to the steady-state conditions. Trend of quadrature axis voltage  $v_q$  across different steady-state velocities is found to be in line with the theory. However, as for the direct axis voltage  $v_d$ , we see a behavior in contrary to theory. In particular, as opposed to theoretical expectation, which is  $v_d \approx 0$  independently of the machine's velocity under steady-state conditions, we observe the strange behavior illustrated in Fig. 2 from both hardware and simulation experiments. This occurs despite FOC sees  $i_d \approx 0$  at its execution steps during steady-state conditions meaning that FOC thinks that the command  $i_d^* = 0$  is tracked perfectly with a non-zero  $v_d$  voltage. Similar to what is observed in [2], inconsistency of the model will inevitably translate to identification results badly in the form of increased estimation error at high speeds. In the next subsection, we try to bring an understanding to this phenomena, which seems to be an inadequacy of existing models compared to experimental results, and propose an extended model accordingly with the hopes of improving parameter estimation performance.

### B. Proposed Model

As seen in Fig. 1, virtual  $dq$  voltages computed by the controller are transformed to  $abc$  frame using a sampled electrical angle. However, for nonzero speeds, that discretized transformation causes a deviation from actual angle over the course of a single control cycle since electrical angle continue to change. Denoting desired  $dq$  voltages by  $v_{dq}^*$  and approximating electrical angle over the course of a single control cycle from time step  $k$  to  $k+1$  with a constant acceleration motion model [16] with acceleration  $\alpha$  corresponding to electrical angle and velocity

$$\begin{aligned}\theta(t) &= \theta_k + \omega_k t + 0.5\alpha_k t^2 \\ \omega(t) &= \omega_k + \alpha_k t\end{aligned}\quad (2)$$

for  $t \in [0, T_s)$ , actual  $dq$  voltages in the same time interval is calculated as

$$\begin{aligned}v_{dq}(t) &= T_{abc}^{dq}(\theta(t)) T_{dq}^{abc}(\theta(0)) v_{dq}^* \\ &= \begin{bmatrix} v_d^* \cos \tilde{\theta}(t) - v_q^* \sin \tilde{\theta}(t) \\ v_q^* \cos \tilde{\theta}(t) - v_d^* \sin \tilde{\theta}(t) \end{bmatrix}\end{aligned}\quad (3)$$

with  $\theta(0) = \theta_k$ ,  $\omega(0) = \omega_k$  and  $\tilde{\theta}(t) := \theta(t) - \theta(0)$  denoting the error in sampled electrical angle. Equation (3) shows that  $dq$  axes become coupled as a result of the ongoing motion in between two samples, and error in applied voltage accumulates with speed and acceleration in practice due to discrete control actions.

Regarding that analysis, we hypothesize that discrepancy of the existing model illustrated in Fig. 2 stems from voltage errors modeled in Eq. (3). To test our hypothesis, we manipulate Eq. 1 by substituting Eq. (3) and Eq. (2) and obtain a time-varying differential equation of the electrical dynamics in the state space form  $\dot{x} = A(t)x + u(t)$  where

$$\begin{aligned}x &= \begin{bmatrix} i_d \\ i_q \end{bmatrix} & A(t) &= \begin{bmatrix} -R/L & \omega(t) \\ -\omega(t) & -R/L \end{bmatrix} \\ u(t) &= \frac{1}{L} \begin{bmatrix} v_d^* \cos \tilde{\theta}(t) - v_q^* \sin \tilde{\theta}(t) \\ v_q^* \cos \tilde{\theta}(t) - v_d^* \sin \tilde{\theta}(t) - \omega(t)\lambda_{pm} \end{bmatrix}.\end{aligned}$$

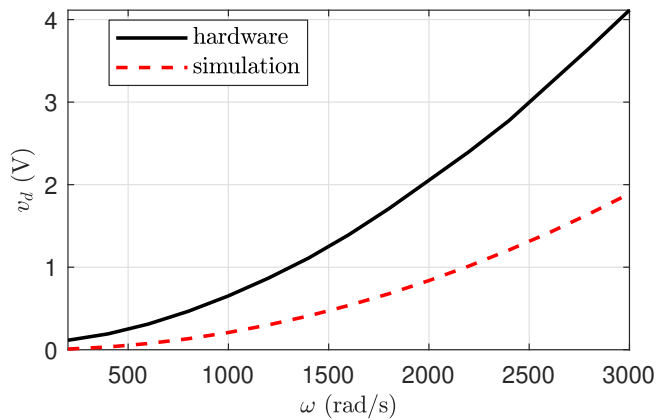


Figure 2: Steady-state values of  $v_d$  under zero command i.e.,  $i_d^{ref} = 0$  at different speeds.

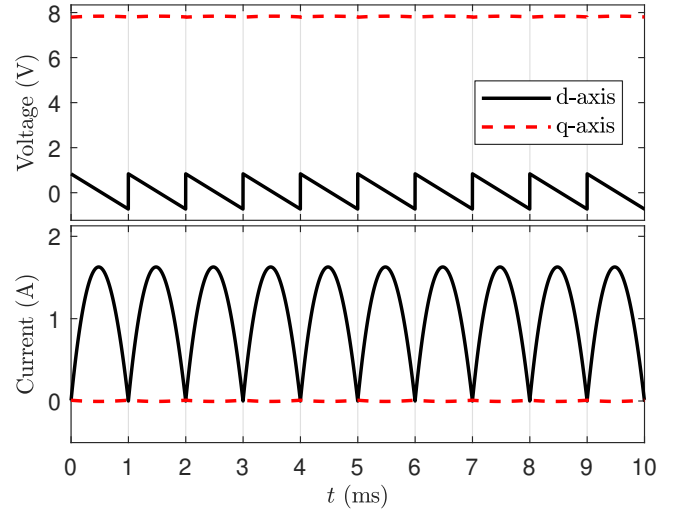


Figure 3: Current (top) and voltages (bottom) in  $dq$  frame, shown with solid black line and dashed red line, respectively, of both the new analytical model and high fidelity numerical simulation coincides perfectly during steady-state conditions of  $\omega = 2000\text{rad/s}$ .

Assuming  $t^2 \approx 0$ , that time-varying dynamical system admits a closed-form analytical solution which can be found by following the standard form of solutions for linear time-varying systems [17]

$$x(t) = e^{At}x(0) + e^{At} \int_0^t e^{-A\tau} u(\tau) d\tau. \quad (4)$$

Here, we omit explicit expression of solutions because of space limitations.

Now that we have a new model of PMSM integrated with a discrete controller, numerical simulation results can be compared to analytical solution Eq. (4) supplied with discrete current measurement and controller output samples as initial conditions for each control cycle. Before proceeding to that, we first check how the model behaves under steady-state conditions. Even though the system dynamics are first-order and simple, they exhibit a strange behavior in agreement with both new analytical model and numerical simulation of dynamics in Eq. (1), as illustrated in Fig. 3, due to voltage deviations explained previously in this section. Aligning with the previous discussion, we see that  $d$  axis current hits zero at every controller execution marked with vertical grid lines but follows an awkward trajectory in between them. Furthermore, comparison of numerical simulations of continuous system dynamics in Eq. (4) to analytical solution reveals that they match in both transient and steady-state phases. That validates both the new model's capabilities and our arguments for explaining the phenomena in Fig. 2 with electrical angle error accumulating over a period due to motor speed.

### III. PARAMETER IDENTIFICATION

The one step analytical solution in Eq. (4) can be equivalently written as a nonlinear function  $h(\Phi)$  of motor parame-

ters  $\Phi := [R, L, \lambda_{pm}]^T$ . In particular, given a parameter guess  $\hat{\Phi}$ , initial conditions, and control actions from samples at  $k^{th}$  time step, the function  $h_k(\hat{\Phi})$  predicts the states at time step  $k + 1$ .

Now, suppose that we would like to improve our parameter estimation vector  $\hat{\Phi}_i$  based on  $N$  measurements collected so far. To this end, we consider the prediction error of current estimations as a function of parameters  $\hat{\Phi}$  as a criteria and define the cost function

$$J_N(\hat{\Phi}) := \frac{1}{2} \sum_{k=1}^N (h_k(\hat{\Phi}) - x_{k+1})^T (h_k(\hat{\Phi}) - x_{k+1}).$$

Based on that cost function, we propose to adopt the damped-Gauss Newton strategy [18] with damping  $\mu$  on a previous guess

$$\hat{\Phi}_{i+1} = \hat{\Phi}_i - \left( \frac{\partial^2 J_N}{\partial \hat{\Phi}^2} \Big|_{\hat{\Phi}_i} + \mu I \right)^{-1} \frac{\partial J_N}{\partial \hat{\Phi}} \Big|_{\hat{\Phi}_i} \quad (5)$$

where cost function, its gradient and Hessian can be computed recursively as

$$\begin{aligned} \frac{\partial J_k}{\partial \hat{\Phi}} &= \frac{\partial J_{k-1}}{\partial \hat{\Phi}} + \left( \frac{\partial h_k}{\partial \hat{\Phi}} \right)^T [h_k(\hat{\Phi}) - x((k+1)T_s)] \\ \frac{\partial^2 J_k}{\partial \hat{\Phi}^2} &= \frac{\partial^2 J_{k-1}}{\partial \hat{\Phi}^2} + \left( \frac{\partial h_k}{\partial \hat{\Phi}} \right)^T \left( \frac{\partial h_k}{\partial \hat{\Phi}} \right). \end{aligned}$$

The damping  $\mu$  in Eq. (5), also known as a regularization term, is used to avoid numerical problems related to possible degeneracy of the Hessian. During implementation, we repeat the Gauss-Newton iteration in Equation (5), after having collected  $N$  samples. Furthermore, the gradient and Hessian are updated with each sample according to recursive rule expressed above in order to minimize computational requirements. The recursive procedure facilitates the computational needs of the algorithm by avoiding multiplication of large matrices which would arise if data were processed in batches. In particular, we would have to deal with vectors and matrices of size proportional to number of samples  $N$  instead of number of motor parameters which is only three. Finally, note that the parameter  $N$  has a filtering effect, thus its optimal choice depend on both signal-to-noise ratio and sampling time  $T_s$ . However, in this paper, we use a fix value for  $N$  as described in Sec. IV.

#### IV. SIMULATION EXPERIMENTS

Systematic simulations of the setup used in Sec. II-A to generate Fig. 2 are conducted to assess the performance of parameter estimation with the proposed model and identification strategy in comparison to the existing model. Our simulation environment involves current sensing noise, quantized encoder outputs, and PWM inverter. The controller runs at frequencies described in Sec. II-A, and Gauss-Newton based parameter identification algorithm runs at 10 Hz corresponding to sample size  $N = 1000$  which are collected at current control execution frequency. The analytical model (4) requires acceleration and velocity feedbacks, which may contain high frequency noise

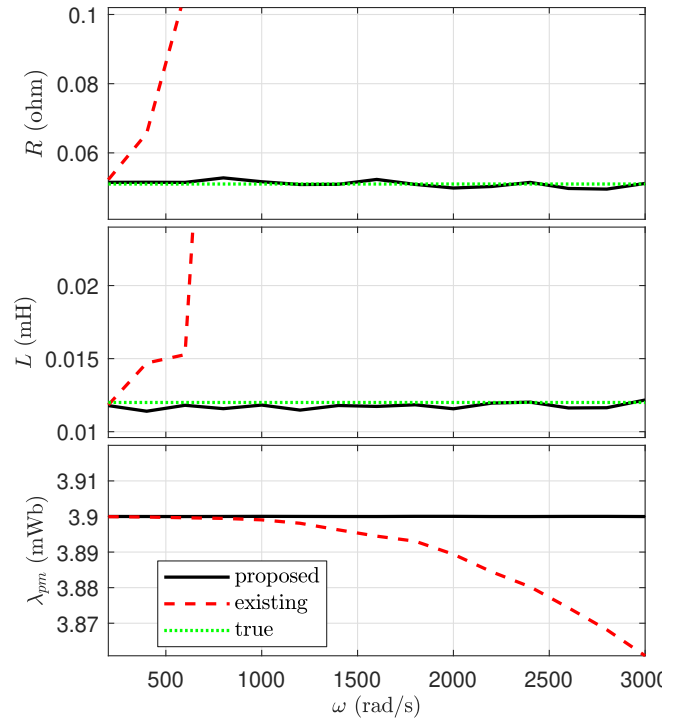


Figure 4: True and estimated values of resistance (top), inductance (middle), and flux linkage (bottom) at different speeds. Resistance and inductance estimates based on conventional models increases exponentially with speed. Axis limits do not cover results of estimation with existing models for visibility purposes.

when computed from quantized position-measurements using finite differences. In order to eliminate those effects, we employ Savitzky-Golay smoother [19] with window length of 21 samples and a third order polynomial. This brings a delay equivalent to 11 sampling period of current controller into parameter estimation results.

First, we evaluate estimation accuracy of each element in  $\Phi$  as a function of velocity. As shown in Fig. 4, the proposed model provides accurate estimates independently of the speed as opposed to existing model identification performance getting significantly worse as speed increases. This performance degradation manifests itself with exponentially increasing errors in resistance and inductance. Estimation based on existing model behaves so since voltage deviations in  $d$  frame increase with speed, hence leading to inaccurate estimation of  $R$  and  $L$ . On the other hand,  $q$  frame dynamics remain consistent with the simple model, and since  $\lambda_{pm}$  appears only in  $q$  frame dynamics, estimation of  $\lambda_{pm}$  is affected only by a little amount as speed changes.

Having shown that the new model prevails over existing models, we now investigate how a parameter estimation using the new model reacts to parameter changes that might occur in an actual system as a result of temperature changes. To that end, we conducted a simulation where true parameters change

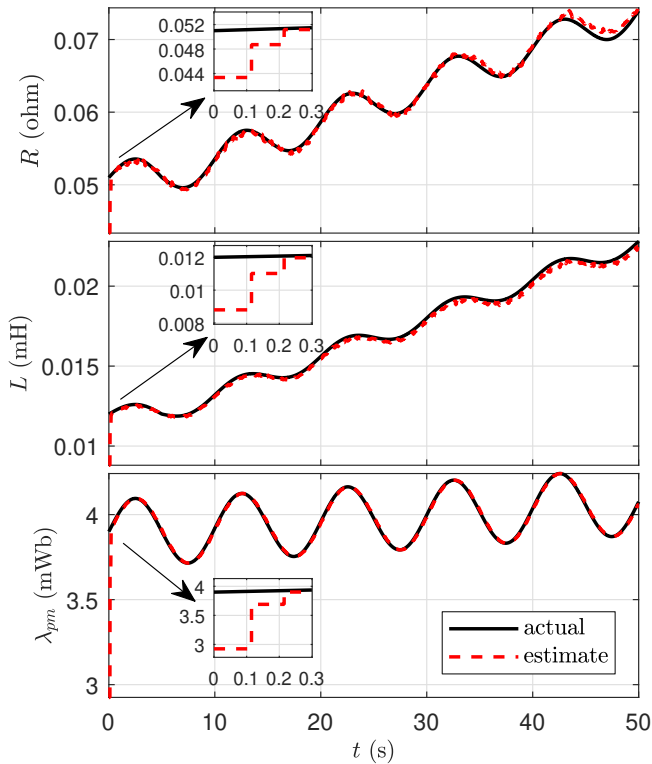


Figure 5: Estimator (dashed red line) tracking time-varying motor parameters (solid black line) with resistance  $R$  (top), inductance  $L$  (middle), and back-emf constant  $\lambda_{pm}$  (bottom) under a constant speed reference of  $\omega = 1000\text{rad/s}$ .

as a superposition of a constant, sinusoid, and ramp according to

$$\Phi(t) = \Phi(0)(1 + a_p \sin(2\pi f_p t) + r_p t)$$

where  $f_p = 0.1\text{Hz}$  and  $a_p$  and  $r_p$  are chosen to be different for each parameter. We find that the proposed estimator can follow dynamic changes in true parameter values at  $0.1\text{Hz}$ . Estimated and true parameters are visualized in Fig. 5, illustrating a capability of the proposed estimator. In order to characterize the upper limit of parameter tracking frequency, we performed additional experiments, which concludes that bandwidth of the proposed estimator is around  $0.5\text{Hz}$  meaning that it can track changes up to  $0.5\text{Hz}$  precisely.

Finally, we evaluate the robustness of the estimator against changes in encoder resolution, PWM frequency, signal-to-noise ratio of current measurements modeled by the standard deviation of the noise, and motor speed. The results are reported in the form of a Karnaugh map in Table I where  $L$  and  $H$  means that corresponding parameter takes a low and high value, respectively. The results show that the algorithm is sufficiently robust for non-mission critical applications such as temperature monitoring. On the other hand, we observe that accuracy degrades intuitively with lower PWM frequency, lower signal-to-noise ratio, lower encoder resolution, and higher motor speeds. measurement noise are

the most important factors. In particular, motor speed and measurement noise have relatively more effect on estimation. Finally, we can say that PWM frequency is not important as long as it is selected above a reasonable threshold, and performance change with encoder resolution is little since a smoothing algorithm is used to obtain position, velocity, and acceleration measurements accurately.

## V. CONCLUSIONS

In this paper, we propose a new predictive model for parameter estimation of PMSMs based on experimental evidence suggesting the use of more sophisticated models for parameter identification particularly in high speed applications. The proposed model covers deficiencies caused by digital control implementations, which are not accounted by existing models. Based on mathematical development, deviations of existing models are explained with error in computed controller voltages during an entire sampling period of control loop due to digital control implementation. Based on analytical solution of the new model, a recursive parameter estimator minimizing a nonlinear optimization problem is presented. The proposed model and associated parameter estimator are verified and shown to outperform an existing model in a detailed simulation study.

Since this is a new model, future work might be conducted in several complementary directions: The proposed model and estimator should be verified on a hardware. The model can be extended to capture slot effects in slotted motors and to include PWM effects by considering its harmonic content up to a reasonable number of harmonics. Even though a Gauss-Newton algorithm is used for identification in this paper, development of statistically optimal identification algorithms (e.g., maximum likelihood estimation or bayesian estimation) will be useful to obtain robust estimators in the presence of high measurement noise and encoder quantization. Furthermore, following statistical work, theoretical performance limits of identification based on the proposed model can be derived (e.g., in terms of Cramer-Rao Lower Bound). Finally, an interesting area of research can be derivation of observability conditions for the new model toward defining input strategies with guaranteed persistence of excitation as an alternative to conventional DC current injection in the literature.

## VI. ACKNOWLEDGEMENT

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Table I: Percentage errors of parameters of the PMSM (resistance, inductance, permanent magnet's flux linkage) during steady-state motion at under different conditions of PWM frequency ( $L : 20kHz$   $H : 50kHz$ ), white noise power corresponding to standard deviations ( $L : 0.045A$   $H : 0.447A$ ), encoder resolutions in terms of pulse per rotation ( $L : 32$   $H : 512$ ), and mechanical speeds ( $L : 200rad/s$   $H : 1200rad/s$ ).

		PWM frequency Noise Power			
		$LL$	$LH$	$HL$	$HH$
Encoder Resolution Motor Speed	$LL$	[0.13 , 1.12 , 0.38]	[0.43 , 1.37 , 0.21]	[0.11 , 1.03 , 0.23]	[0.56 , 1.31 , 0.05]
	$LH$	[0.98 , 2.22 , 0.05]	[1.31 , 2.68 , 0.05]	[0.72 , 2.18 , 0.10]	[1.08 , 2.79 , 0.10]
	$HL$	[0.01 , 0.39 , 0.13]	[0.02 , 0.33 , 0.13]	[0.01 , 0.48 , 0.15]	[0.14 , 0.46 , 0.13]
	$HH$	[0.76 , 0.69 , 0.03]	[1.08 , 0.42 , 0.03]	[0.91 , 0.67 , 0.03]	[0.93 , 0.45 , 0.03]

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