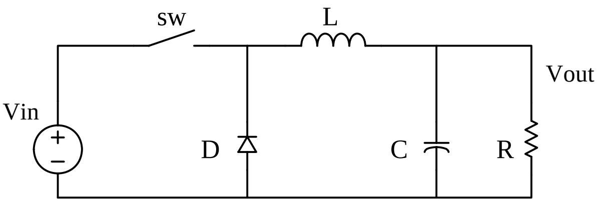
Exercise 1 solution

Problem 1

$Vin := 10V$ $\qquad f_{sw} := 125kHz$ $\qquad C := 100\mu F$

$D := 0.6$ $\qquad R_L := 15\Omega$

a) Buck



Find inductance L_{min} for CCM:

$Vo := D \cdot Vin$ $\qquad Vo = 6\,V$

$Io := \frac{Vo}{R_L}$ $\qquad Io = 0.4\,A$

Output current equals to inductor average current Iav.
Minimum inductance for CCM (more precisely Critical CCM) can be found the following way:

$\Delta I = 2 \cdot Iav = 2 \cdot Io$ $\qquad \Delta I = \frac{(Vin - Vo) \cdot D}{L \cdot f_{sw}}$

$L_{min} := \frac{(Vin - Vo) \cdot D}{2 \cdot Io \cdot f_{sw}}$ $\qquad L_{min} = 24 \cdot \mu H$

$L := 5 \cdot L_{min}$ $\qquad L = 120 \cdot \mu H$

$Po := Vo \cdot Io$ $\qquad Po = 2.4\,W$

$\Delta I := \frac{(Vin - Vo) \cdot D}{L \cdot f_{sw}}$ $\qquad \Delta I = 0.16\,A$

$I_{Lav} := Io$ $\qquad I_{Lav} = 0.4\,A$

$I_{Dav} := Io \cdot (1 - D)$ $\qquad I_{Dav} = 0.16\,A$

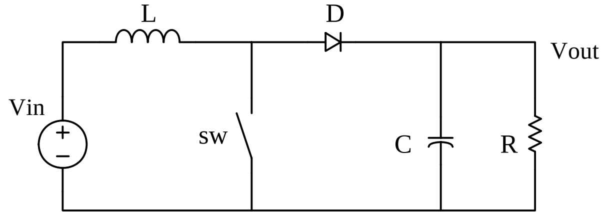
For triangular waveform the RMS current :

$$I_{Lrms} := \sqrt{\left(I_{Lav} - \frac{\Delta I}{2}\right) \cdot \left(I_{Lav} + \frac{\Delta I}{2}\right) + \frac{\Delta I^2}{3}} \qquad I_{Lrms} = 0.403\,A$$

$$I_{Lrms2} := \sqrt{I_{Lav}^{\;2} + \frac{\left(\frac{\Delta I}{2}\right)^2}{3}} \qquad I_{Lrms2} = 0.403\,A$$

$V_{Dmax} = Vin$ $\qquad V_{SWmax} = Vin$

b) Boost

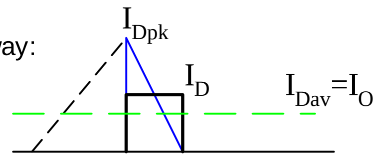


Find inductance L_{min} for CCM:

$$Vo := Vin \cdot \frac{1}{1 - D}$$ $Vo = 25\,V$

$$Io := \frac{Vo}{R_L}$$ $Io = 1.667\,A$

Output current equals to diode average current I_{Dav}.
Minimum inductance for CCM can be found the following way:



$$\Delta I = 2 \cdot I_D = \frac{2 \cdot Io}{1 - D} \qquad \Delta I = \frac{Vin \cdot D}{L \cdot f_{sw}}$$

$$L_{min} := \frac{Vin \cdot D \cdot (1 - D)}{2 \cdot Io \cdot f_{sw}}$$ $L_{min} = 5.76 \cdot \mu H$

$$L := 5 \cdot L_{min}$$ $L = 28.8 \cdot \mu H$

$$Po := Vo \cdot Io$$ $Po = 41.667\,W$

$$\Delta I := \frac{Vin \cdot D}{L \cdot f_{sw}}$$ $\Delta I = 1.667\,A$

$$I_{Lav} := \frac{Io}{1 - D}$$ $I_{Lav} = 4.167\,A$

$$I_{Dav} := Io$$ $I_{Dav} = 1.667\,A$

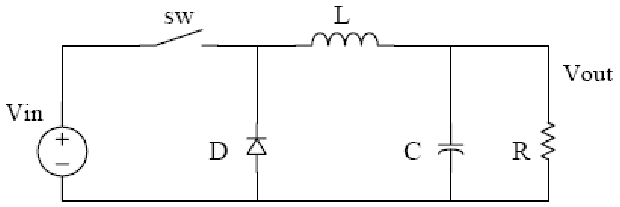
For triangular waveform the RMS current :

$$I_{Lrms} := \sqrt{\left(I_{Lav} - \frac{\Delta I}{2}\right) \cdot \left(I_{Lav} + \frac{\Delta I}{2}\right) + \frac{\Delta I^2}{3}}$$ $I_{Lrms} = 4.194\,A$

$$V_{Dmax} = Vo \qquad V_{SWmax} = Vo$$

Problem 2

Buck converter:



Current waveform for Discontinuous Mode:

For the steady state operation the average current through the inductor L is constant and average voltage
equals zero.

$$(Vin - Vo) \cdot D_{on} = Vo \cdot D_f$$

D_f is discharge part of the period.

$$\frac{Vo}{Vin} = \frac{D_{on}}{D_{on} + D_f}$$

(When D_f=1-D_{on} we have "regular" formula for Buck converter)

$$\Delta I = \frac{Vo \cdot D_f}{L \cdot f_{sw}} \quad => \quad D_f = \frac{L \cdot f_{sw} \cdot \Delta I}{Vo}$$

Output current equals to the average inductor current:

$$Io = I_{Lav} = \frac{\Delta I}{2} \cdot \left(D_{on} + D_f\right) = \frac{Vo}{R_L}$$

R_L - load resistance

$$\Delta I = \frac{2\,Vo}{R_L} \cdot \frac{1}{\left(D_{on} + D_f\right)}$$

$$D_f = \frac{L \cdot f_{sw} \cdot 2}{R_L \cdot \left(D_{on} + D_f\right)}$$

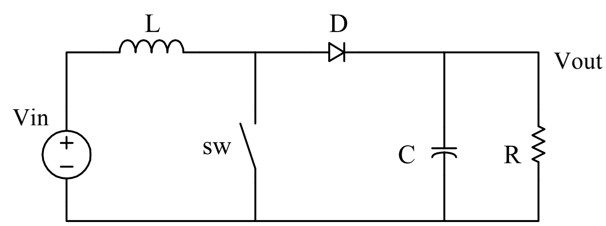
$$D_f^2 + D_{on} \cdot D_f - \frac{2\, L \cdot f_{sw}}{R_L} = 0$$

Solving this equation we can find D_f:

$$D_f = \sqrt{\frac{D_{on}^2}{4} + 2 \cdot \frac{L \cdot f_{sw}}{R_L}} - \frac{D_{on}}{2}$$

$$\frac{Vo}{Vin} = \frac{D_{on}}{D_{on} + \sqrt{\frac{D_{on}^2}{4} + 2 \cdot \frac{L \cdot f_{sw}}{R_L}} - \frac{D_{on}}{2}} = \frac{D_{on}}{\frac{D_{on}}{2} + \sqrt{\frac{D_{on}^2}{4} + 2 \cdot \frac{L \cdot f_{sw}}{R_L}}}$$

Boost converter:



Average voltage on the inductor L is zero:

$$Vin \cdot D_{on} = (Vo - Vin) \cdot D_f$$

Input power equals to the output power (supposing the losses are neglible).

$$Pin = Vin \cdot Iin_{av} = Pout = \frac{Vo^2}{R_L}$$

$$Iin_{av} = \frac{\Delta I}{2} \cdot \left(D_{on} + D_f\right)$$

$$\Delta I = \frac{Vin \cdot D_{on}}{L \cdot f_{sw}}$$

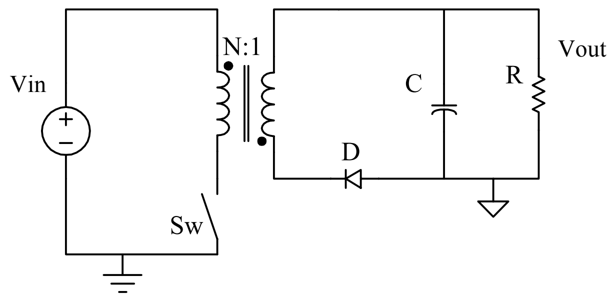
$$Pin = Vin \cdot \frac{Vin \cdot D_{on}}{2 \cdot L \cdot f_{sw}} \cdot \left(D_{on} + D_f\right)$$

$$\left|\begin{array}{l} \frac{Vin^2 \cdot D_{on}}{2 \cdot L \cdot f_{sw}} \cdot \left(D_{on} + D_f\right) = \frac{Vo^2}{R_L} \\ Vin \cdot D_{on} = (Vo - Vin) \cdot D_f \end{array}\right.$$

Solving the previous set of two equations for Vo, we have:

$$\frac{Vo}{Vin} = \frac{1}{2}\cdot\left(1 + \sqrt{1 + \frac{2\cdot D_{on}^{2}\cdot R_L}{L\cdot f_{sw}}}\right)$$

Flyback converter:



Average voltage on the inductor L is zero:

$$Vin\cdot D_{on} = Vo\cdot N\cdot D_f$$

$$Pout = \frac{Vo^2}{R_L} = \frac{\Delta I_s^{2}\cdot L_s}{2}\cdot f_{sw}$$

$$\Delta I_s = \frac{Vo\cdot D_f}{L_s\cdot f_{sw}}$$

$$\frac{Vo^2}{R_L} = \left(\frac{Vo\cdot D_f}{L_s\cdot f_{sw}}\right)^2\cdot\frac{L_s}{2}\cdot f_{sw} \qquad => \qquad D_f^{2} = \frac{2\,L_s\cdot f_{sw}}{R_L}$$

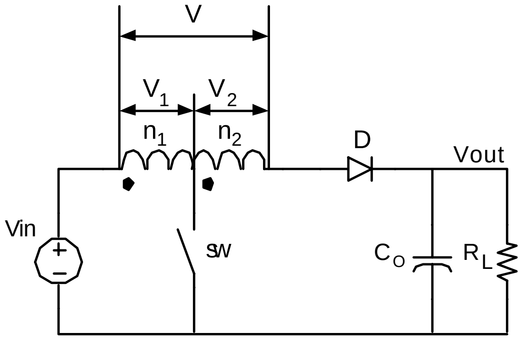
Substituting in the first equation:

$$Vin\cdot D_{on} = Vo\cdot N\cdot\sqrt{\frac{2\,L_s\cdot f_{sw}}{R_L}} = Vo\cdot N\cdot\sqrt{\frac{2\,L\cdot f_{sw}}{R_L\cdot N^2}}$$

$$\frac{Vo}{Vin} = D_{on}\cdot\sqrt{\frac{R_L}{2\,L\cdot f_{sw}}}$$

Problem 3:

Tapped boost converter:



$$\frac{V}{n} = \frac{d\phi}{dt}$$

Since the magnetic flux must maintain continuous, we obtain:

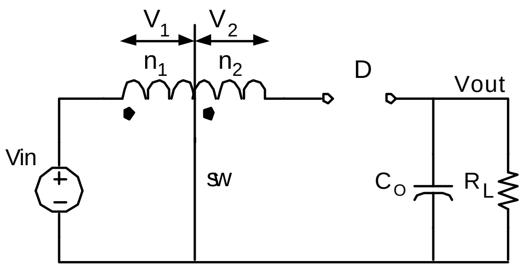
$$\frac{V}{n_1 + n_2} = \frac{V_1}{n_1} \quad \text{------>} \quad \frac{V}{V_1} = \frac{n_1 + n_2}{n_1} = n$$

Separate the operation into two sub-intervals: ON and OFF

ON sub-interval:

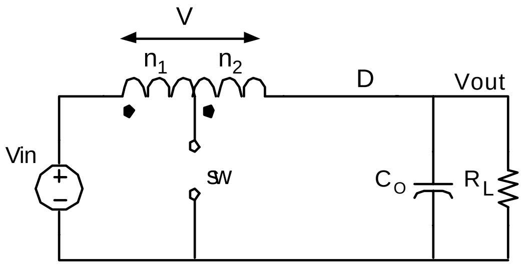
$$V_1 = V_{in}$$

$$V = V_{in}\left(\frac{n_1 + n_2}{n_1}\right)$$



OFF sub-interval:

$$V = V_{in} - V_{out}$$



The average voltage <V> of the inductor L must be zero for every switching cycle.

$$V_{in} \cdot D\left(\frac{n_1 + n_2}{n_1}\right) + \left(V_{in} - V_{out}\right)(1 - D) = 0$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 - D} \cdot \left(1 + D \cdot \frac{n_2}{n_1}\right)$$