

Exercise 2 solution

Problem 1

$$a_1 := 0.5 \text{ cm} \quad a_2 := 1 \text{ cm} \quad a_3 := 2 \text{ cm}$$

$$\mu r_1 := 50 \quad \mu r_2 := 100 \quad \mu r_3 := 200$$

$$I_{pk1} := 0.4 \text{ A} \quad I_{pk2} := 4 \text{ A}$$

$$L_1 := 4 \text{ mH} \quad L_2 := 600 \text{ } \mu\text{H}$$

$$f_{sw} := 100 \text{ kHz} \quad J := 4.5 \frac{\text{A}}{\text{mm}^2} \quad B_{sat} := 0.3 \text{ T} \quad K := 0.5$$

$$\mu_0 := 4 \cdot \pi \cdot 10^{-7} \frac{\text{H}}{\text{m}} \quad i := 1..3 \quad mT := 10^{-3} \text{ T}$$

The magnetic core parameters are:

$$l_e = 2 \cdot \pi \cdot r_e = 2 \cdot \pi \cdot \frac{r_{in} + r_{out}}{2}$$

$$l_{e_i} := 2 \cdot \pi \cdot \frac{a_i + 2 \cdot a_i}{2} \quad l_{e_1} = 47.124 \text{ mm} \quad l_{e_2} = 94.248 \text{ mm} \quad l_{e_3} = 188.496 \text{ mm}$$

$$A_{e_i} := (a_i)^2 \quad A_{e_1} = 25 \text{ mm}^2 \quad A_{e_2} = 100 \text{ mm}^2 \quad A_{e_3} = 400 \text{ mm}^2$$

$$A_{w_i} := \pi \cdot (a_i)^2 \quad A_{w_1} = 78.54 \text{ mm}^2 \quad A_{w_2} = 314.159 \text{ mm}^2 \quad A_{w_3} = 1.257 \cdot 10^3 \text{ mm}^2$$

$$AP_i := A_{e_i} \cdot A_{w_i} \quad AP_1 = 1.96 \cdot 10^3 \text{ mm}^4 \quad AP_2 = 3.142 \cdot 10^4 \text{ mm}^4 \quad AP_3 = 5.027 \cdot 10^5 \text{ mm}^4$$

$$Vol_i := A_{e_i} \cdot l_{e_i} \quad Vol_1 = 1.18 \cdot 10^3 \text{ mm}^3 \quad Vol_2 = 9.425 \cdot 10^3 \text{ mm}^3 \quad Vol_3 = 7.54 \cdot 10^4 \text{ mm}^3$$

For small ΔI (0.2 I_{pk}) I_{rms} approx. equals to Iaverage.

Skin depth is:

$$S := \frac{76}{\sqrt{f_{sw}}} \cdot \text{mm} \sqrt{\text{Hz}} \quad S = 0.24 \text{ mm}$$

1.1

Area Product needed for the first inductor (taking $B_{\max} := 0.25 \text{ T}$):

$$\frac{L_1 \cdot (I_{pk1})^2}{J \cdot K \cdot B_{\max}} = 1.138 \cdot 10^3 \text{ mm}^4 \quad \Rightarrow \text{ the smallest core } a = 0.5 \text{ cm}$$

Check the maximum number of turns in the window of chosen toroid:

$$I_{rms1} = I_{av} = I_{pk1} - \frac{\Delta I_1}{2} = 0.9 \cdot I_{pk1}$$

$$I_{rms1} := 0.9 \cdot I_{pk1}$$

$$I_{rms1} = 0.36 \text{ A}$$

Needed winding area for inductor RMS current :

$$A_{cu} := \frac{I_{rms1}}{J}$$

$$A_{cu} = 0.08 \text{ mm}^2$$

$$\text{strands} := \text{ceil} \left(\frac{A_{cu}}{\pi \cdot S^2} \right)$$

$$\text{strands} = 1$$

No need for a litz wire.

$$d_{cu} := \sqrt{\frac{4 \cdot A_{cu}}{\pi \cdot \text{strands}}}$$

$$d_{cu} = 0.319 \text{ mm}$$

Winding: one strand of copper wire $d_{cu} = 0.319 \text{ mm}$

Maximum number of turns:

$$n_{\max} := \frac{A_{w1}}{\pi \cdot \left(\frac{d_{cu}}{2} \right)^2 \cdot \text{strands}} \cdot K$$

$$n_{\max} = 490.874$$

Choose μ_r such that $n < n_{\max}$ and $B_{\max} < 0.25 \text{ T}$:

$$n_1 = \sqrt{\frac{L_1 \cdot l_e}{\mu_r \cdot \mu_0 \cdot A_e}}$$

$$n_{1_i} := \text{round} \left[\sqrt{\frac{L_1 \cdot l_{e1}}{\mu_{r_i} \cdot \mu_0 \cdot A_{e1}}} \right]$$

$$n_{1_1} = 346$$

$$n_{1_2} = 245$$

$$n_{1_3} = 173$$

$$B_{\max_1} := \frac{L_1 \cdot I_{pk1}}{A_{e1} \cdot n_{1_1}}$$

$$B_{\max_1} = 0.185 \text{ T}$$

$$B_{\max_2} = 0.261 \text{ T}$$

$$B_{\max_3} = 0.37 \text{ T}$$

$B_{\max} := B_{\max_1}$ is acceptable and $\mu_r = 50$, number of turns $n_1 := 212$

The following core is chosen: $a_1 = 0.5 \text{ cm}$ $\mu_{r_1} = 50$

$$\Delta B = B_{\max} \cdot \frac{\Delta I}{I_{pk}} = B_{\max} \cdot 0.2$$

$$\Delta B := 0.2 \cdot B_{\max}$$

$$\Delta B = 37 \text{ mT}$$

1.2

Area Product needed for the second inductor (taking $B_{\max} := 0.25 \text{ T}$):

$$\frac{L_2 \cdot (I_{pk_2})^2}{J \cdot K \cdot B_{\max}} = 1.707 \cdot 10^4 \text{ mm}^4 \quad \Rightarrow \text{the smallest core } a = a_2 = 1 \text{ cm}$$

Check the maximum number of turns in the window of chosen toroid:

$$I_{rms_2} = I_{av} = I_{pk_2} \cdot \frac{\Delta I_2}{2} = 0.9 \cdot I_{pk_2}$$

$$I_{rms_2} := 0.9 \cdot I_{pk_2}$$

$$I_{rms_2} = 3.6 \text{ A}$$

Needed winding area for inductor RMS current :

$$A_{cu} := \frac{I_{rms_2}}{J}$$

$$A_{cu} = 0.8 \text{ mm}^2$$

$$\text{strands} := \text{ceil} \left(\frac{A_{cu}}{\pi \cdot S^2} \right)$$

$$\text{strands} = 5$$

Litz wire needed !

$$d_{cu} := \sqrt{\frac{4 \cdot A_{cu}}{\pi \cdot \text{strands}}}$$

$$d_{cu} = 0.451 \text{ mm}$$

Winding: ten strands of copper wire $d_{cu} := 0.5 \text{ mm}$

Maximum number of turns:

$$n_{\max} := \frac{A_{w_2}}{\pi \cdot \left(\frac{d_{cu}}{2} \right)^2 \cdot \text{strands}} \cdot K$$

$$n_{\max} = 160$$

Choose μ_r such that $n < n_{\max}$ and $B_{\max} < 0.25 \text{ T}$:

$$n_2 = \sqrt{\frac{L \cdot I_e}{\mu_r \cdot \mu_0 \cdot A_e}}$$

$$n_{2_i} := \text{round} \left[\sqrt{\frac{L_2 \cdot I_{e_2}}{\mu_{r_i} \cdot \mu_0 \cdot A_{e_2}}} \right]$$

$$n_{2_1} = 95$$

$$n_{2_2} = 67$$

$$n_{2_3} = 47$$

$$B_{\max_i} := \frac{L_2 \cdot I_{pk_2}}{A_{e_2} \cdot n_{2_i}}$$

$$B_{\max_1} = 0.253 \text{ T}$$

$$B_{\max_2} = 0.358 \text{ T}$$

$$B_{\max_3} = 0.511 \text{ T}$$

The flux density is too big, the core with $a = a_2 = 1 \text{ cm}$ is not suitable. Next try - core with $a = a_3 = 2 \text{ cm}$

Maximum number of turns:

$$n_{\max} := \frac{A_w \cdot K}{\pi \cdot \left(\frac{d_{cu}}{2}\right)^2 \cdot \text{strands}}$$

$$n_{\max} = 640$$

Choose μ_r such that $n < n_{\max}$ and $B_{\max} < 0.25 \text{ T}$:

$$n_2 = \sqrt{\frac{L \cdot I_e}{\mu_r \cdot \mu_0 \cdot A_e}}$$

$$n_{2_i} := \text{round} \left[\sqrt{\frac{L_2 \cdot I_{e_3}}{\mu_{r_i} \cdot \mu_0 \cdot A_{e_3}}} \right]$$

$$n_{2_1} = 67$$

$$n_{2_2} = 47$$

$$n_{2_3} = 34$$

$$B_{\max_i} := \frac{L_2 \cdot I_{pk_2}}{A_{e_3} \cdot n_{2_i}}$$

$$B_{\max_1} = 0.09 \text{ T}$$

$$B_{\max_2} = 0.128 \text{ T}$$

$$B_{\max_3} = 0.176 \text{ T}$$

$B_{\max} := B_{\max_3}$ is acceptable and $\mu_r = 150$ number of turns $n_2 := 39$

The following core is chosen: $a_3 = 2 \text{ cm}$ $\mu_{r_3} = 200$ (this core is too big, but the only possible)

$$\Delta B = B_{\max} \cdot \frac{\Delta I}{I_{pk}} = B_{\max} \cdot 0.2$$

$$\Delta B := 0.2 \cdot B_{\max}$$

$$\Delta B = 35.3 \text{ mT}$$

Problem 2

$$2.1 \quad V_{in} := 12 \text{ V} \quad V_{out} := 48 \text{ V} \quad I_{out} := 1 \text{ A} \quad f_{sw} := 50 \text{ kHz} \quad J := 4.5 \frac{\text{A}}{\text{mm}^2}$$

$$B_{max} := 0.2 \text{ T} \quad K := 0.5$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{1-D} \quad D := 1 - \frac{V_{in}}{V_{out}} \quad D = 0.75$$

$$I_{Lav} := \frac{I_{out}}{1-D} \quad \Delta I := 0.1 \cdot I_{Lav} \quad I_{Lav} = 4 \text{ A} \quad \Delta I = 0.4 \text{ A}$$

$$L := \frac{V_{in} \cdot D}{\Delta I \cdot f_{sw}} \quad L = 4.5 \cdot 10^{-4} \text{ H}$$

$$I_{pk} := I_{Lav} + \frac{\Delta I}{2} \quad I_{pk} = 4.2 \text{ A}$$

$$I_{rms} := \sqrt{I_{Lav}^2 + \left(\frac{\Delta I}{12}\right)^2} \quad I_{rms} = 4 \text{ A}$$

$$S := \frac{76}{\sqrt{f_{sw}}} \text{ mm} \cdot \sqrt{\text{Hz}} \quad S = 3.399 \cdot 10^{-4} \text{ m}$$

2.2

First find area product to choose the core

$$AP := \frac{L \cdot I_{pk} \cdot I_{rms}}{J \cdot K \cdot B_{max}} \quad AP = 1.68 \cdot 10^{-8} \text{ m}^4$$

The core that satisfies the needed area product is E41/17/12.

$$A_e := 149 \text{ mm}^2 \quad A_w := 114 \text{ mm}^2 \quad A_e \cdot A_w = 1.699 \cdot 10^4 \text{ mm}^4$$

Number of turns:

$$N := \frac{L \cdot I_{pk}}{A_e \cdot B_{max}} \quad N = 63.423$$

Rounding turns number to $N := 64$

$$A_{cu} := \frac{I_{rms}}{J} \quad A_{cu} = 0.889 \text{ mm}^2$$

$$\text{strands} := \text{ceil} \left(\frac{A_{cu}}{\pi \cdot S^2} \right) \quad \text{strands} = 3$$

$$d_{\text{cu}} := \sqrt{\frac{4 \cdot A_{\text{cu}}}{\pi \cdot \text{strands}}}$$

$$d_{\text{cu}} = 0.614 \text{ mm}$$

Winding: Three strands of copper wire $d_{\text{cu}} := 0.5 \text{ mm}$

Window area occupied by the winding:

$$\text{Window} := \pi \cdot \left(\frac{d_{\text{cu}}}{2}\right)^2 \cdot \text{strands} \cdot \frac{N}{K}$$

$$\text{Window} = 75.398 \text{ mm}^2$$

Air gap:

$$l_{\text{g}} := \frac{\mu_0 \cdot N^2 \cdot A_{\text{e}}}{L}$$

$$l_{\text{g}} = 1.704 \text{ mm}$$

Air gap $l_{\text{g}} = 1.704 \text{ mm}$ in the middle leg of E core, or two gaps $\frac{l_{\text{g}}}{2} = 0.852 \text{ mm}$ in the side legs.

$$A_{\text{cu}} \cdot N = 56.891 \text{ mm}^2$$

$$l_{\text{g}} := \frac{\mu_0 \cdot N^2 \cdot A_{\text{e}}}{L}$$

$$l_{\text{g}} = 1.704 \text{ mm}$$

פתרון שאלה מס' 3

נתונים:

$$f_s = 50 \text{ kHz}; \quad P = 200 \text{ W}; \quad V_{in} = \pm 50 \text{ V}; \quad V_{out} = \pm 150 \text{ V}; \\ \Delta B = 0.15 \text{ T}; \quad k = 0.6; \quad J = 4 \times 10^6 \text{ A/m}^2;$$

נתחיל בנוסחת A_p :

$$A_p = A_e \cdot A_w = \frac{\{V_1, t_{on}\} \cdot 2 \cdot I_1(\text{RMS})}{\Delta B \cdot J \cdot k}$$

נחשב את הפרמטרים החסרים:

$$I_1(\text{RMS}) = \frac{P}{V_{in}} = \frac{200}{150} = 1.333 \text{ A};$$

$$T_s = 20 \mu\text{s}; \quad t_{on} = 10 \mu\text{s}; \quad \{V_1, t_{on}\} = 10 \mu \cdot 150 = 1.5 \text{ m}$$

$$A_p = \frac{1.5 \text{ m} \cdot 2 \cdot 1.333}{0.15 \cdot 4 \times 10^6 \cdot 0.6} = 11.11 \text{ n [m}^4]$$

הגוף המתאים ETD34 עם הפרמטרים:

	A_e [mm ²]	l_e [mm]	A_w [mm ²]	V_e [mm ³]	A_p [m ⁴]
ETD 34	97.1	78.6	123	7640	11.94n

נחזור ונחשב את מספר הליפופים ע"פ ה- A_e האמיתי של הגוף ETD34:

$$n_1 = \frac{\{V_1, t_{on}\}}{\Delta B \cdot A_e} = \frac{1.5 \text{ m [V} \cdot \text{sec]}}{0.15 \text{ [T]} \cdot 97.1 \times 10^{-6} \text{ [m}^2]} \approx 103 \text{ [Turns]}$$

$$n_2 = \frac{V_{out}}{V_{in}} n_1 = \frac{1}{3} n_1 \approx 35 \text{ [Turns]}$$

ועכשיו נקבע את עובי החוט:

$$I_1(\text{RMS}) = 1.333 \text{ A}; \quad I_2(\text{RMS}) = 3 \cdot 1.333 = 4 \text{ A};$$

$$A_{Cu1} = \frac{1.333 \text{ [A]}}{4 \text{ [A/mm}^2]} = 0.33 \text{ [mm}^2]; \quad A_{Cu2} = \frac{4 \text{ [A]}}{4 \text{ [A/mm}^2]} = 1 \text{ [mm}^2];$$

לא נשכח להתחשב בתופעת ה-Skin effect:

$$\delta[\text{mm}] \approx \frac{72}{\sqrt{f_s[\text{Hz}]}} = \frac{72}{\sqrt{50\text{k}}} = 0.322 \text{ [mm]}$$

שטח חתך וקוטר יעיל מרבי של חוט עגול:

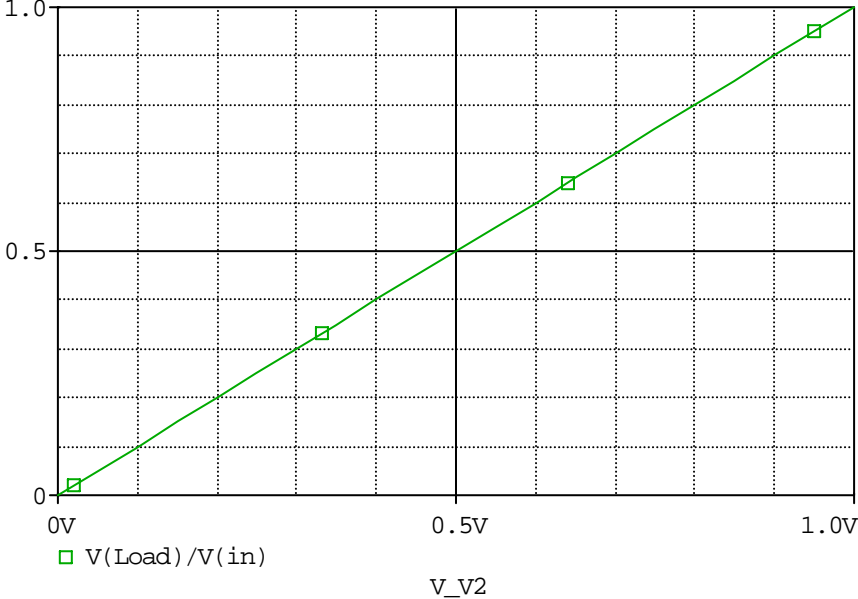
$$S_{\max} = \pi r^2 = \pi \delta^2 \approx 0.326 \text{ [mm}^2]; \quad d_{\max} = 2 \cdot \sqrt{\frac{S_{\max}}{\pi}} = 0.644 \text{ [mm];}$$

$$m_1 = \frac{A_{Cu1}}{S_{\max}} = \frac{0.33}{0.326} \approx 1; \quad m_2 = \frac{A_{Cu2}}{S_{\max}} = \frac{1}{0.326} \approx 3;$$

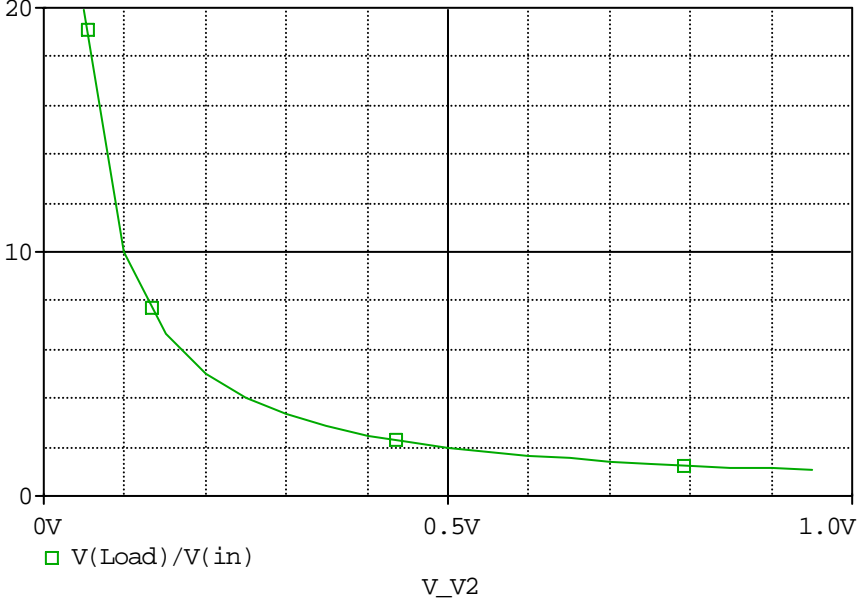
נשתמש בחוט בעל קוטר הקרוב ביותר ל- d_{\max} וזה חוט מהסוג: SWG-22 או 28 mil או בקוטר של 0.711 mm. בראשוני נצטרך חוט יחיד ובמשני חוט ליצה המורכב משלושה חוטים מבודדים.

Problem 4

1) Buck transfer function, duty cycle is a parameter:

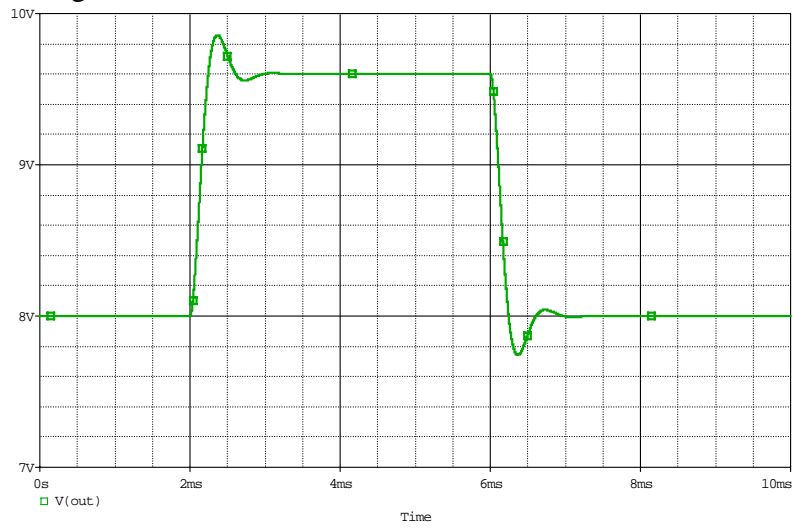


1) Boost transfer function, duty cycle is a parameter:

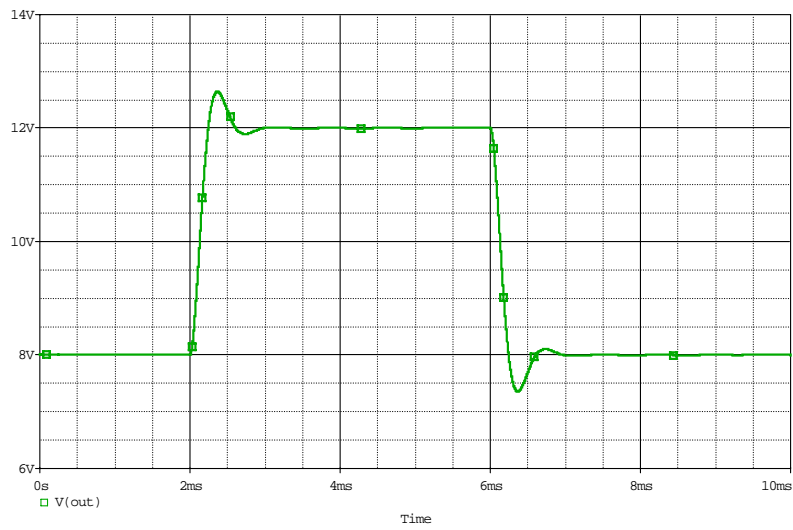


2) Buck

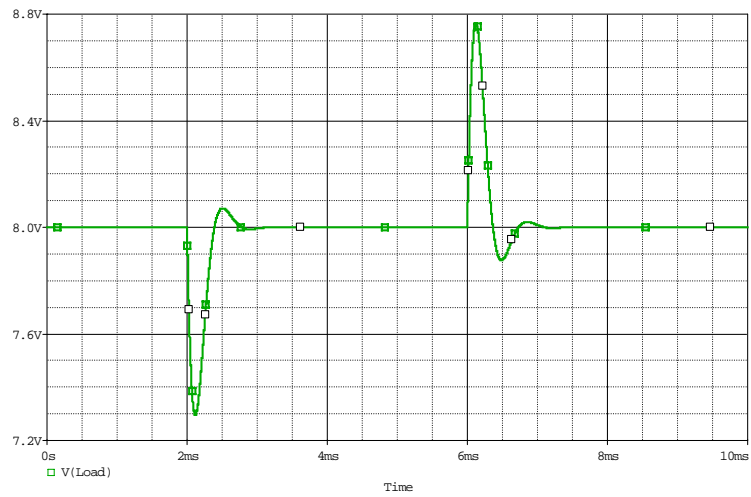
a) Input voltage change of 20%:



b) Duty Cycle change of 0.2:

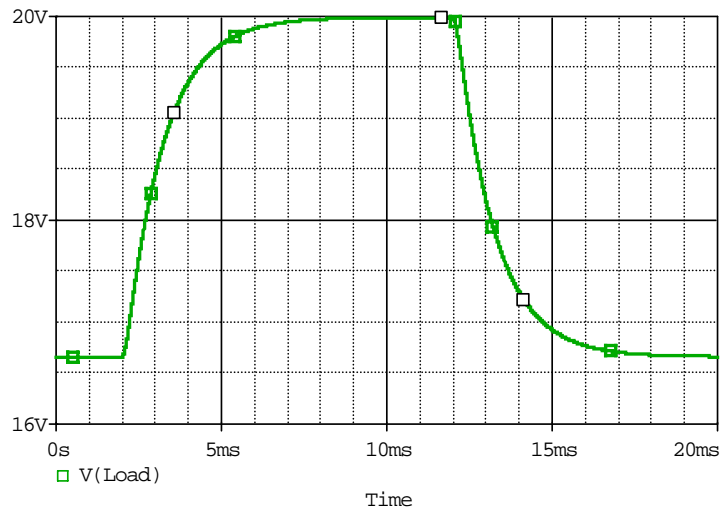


c) Load change of 15%:

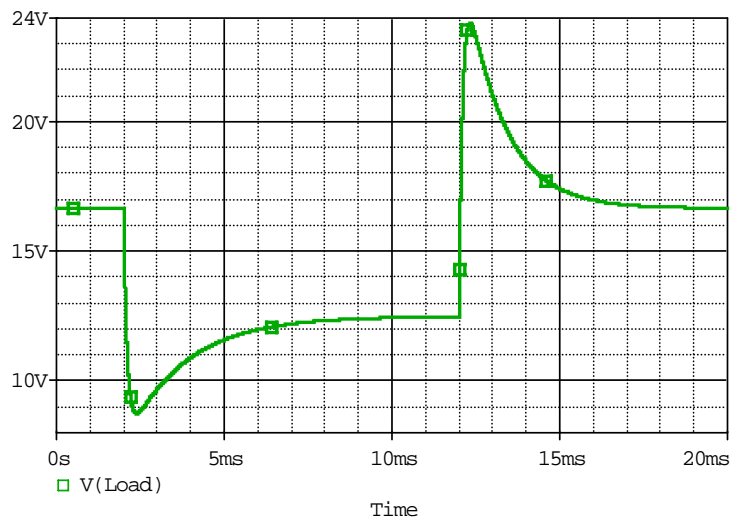


2) Boost

a) Input voltage change of 20%:



b) Duty Cycle change of 0.2:



c) Load change of 15%:

