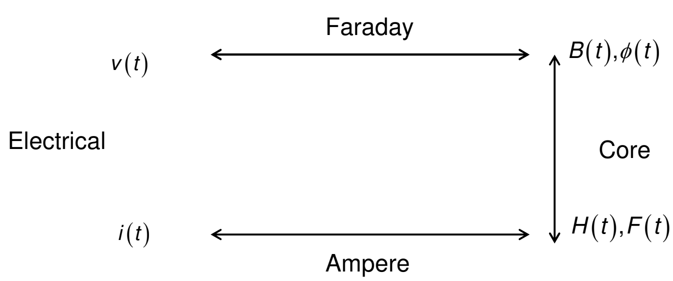
Magnetics

- Faraday's and Amper's laws
- Permeability
- Inductor
  - Reluctance model
  - Air gap
  - Current crowding
  - Inductor design
- Skin effect, proximity effect
- Losses
- Transformer
  - Ideal transformer
  - Real transformer
  - Transformer design

Important relationships



Magnetic quantities
Analogies to electrical quantities

Magnetic field *H*

ℓ

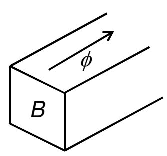
MMF $F_M = H \cdot \ell$

Electric field *E*

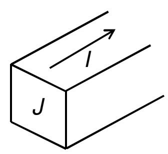
ℓ

Voltage $V = E \cdot \ell$

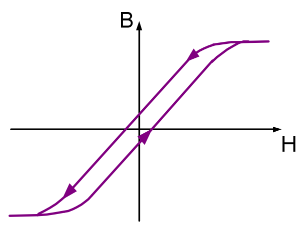
B, ϕ

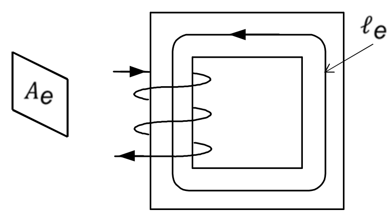


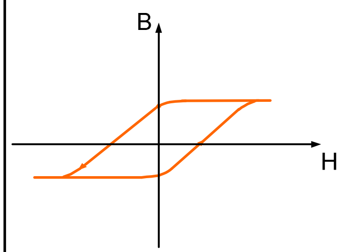
J, I



Important relationships







Faraday's law

$$V = n\frac{d\Phi}{dt} = nA_e\frac{dB}{dt}; \qquad \mu = \frac{\Delta B}{\Delta H}$$

Amper's law

$$\oint H d\ell = n \cdot I \qquad H = \frac{nI}{\ell_e}$$

Units

Φ - magnetic flux Weber [Wb]

V - voltage [V]

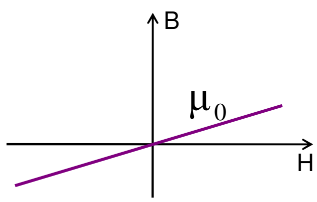
B - flux density $\frac{Wb}{m^2}$ = Tesla [T]

Gauss [G] 1T = 10,000 G

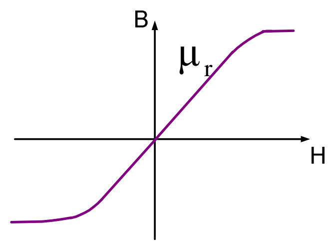
H - magnetic field [A/m]

μ - magnetic permeability [H/m]

Permeability



$$\mu = \frac{\Delta B}{\Delta H}$$



Free space permeability (Vacuum)

$$\mu_0 = 4\pi \cdot 10^{-7}$$

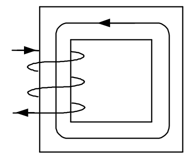
Typical B_{sat}
Ferrite: 0.2 – 0.5 T
Iron powder: 0.5 – 1 T

Typical magnetic material permeability

$$\mu_r = 10^3 - 10^5$$

What is inductance ?

$$V = n\frac{d\phi}{dt} \qquad V = L\frac{dI}{dt}$$



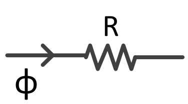
Reluctance model

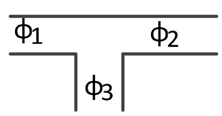
$$F = H\ell_e$$

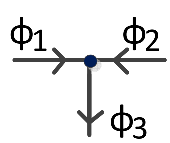
$$H = \frac{B}{\mu} \qquad B = \frac{\varphi}{A}$$

$$R = \frac{\ell_e}{\mu A} \quad \left[H^{-1}\right]$$

$$F = \varphi R$$



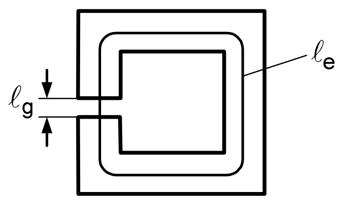




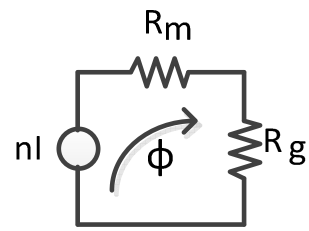
$$\phi_3 = \phi_1 + \phi_2$$

Inductance with gap

$$R_m = \frac{\ell_e}{\mu_r A_e}$$



$$R_g = \frac{\ell_g}{\mu_0 A_e}$$



$$nI = \phi\left(R_m + R_g\right)$$

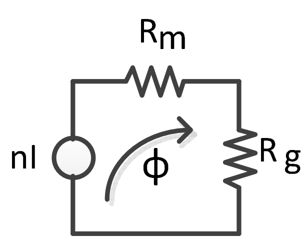
$$v = n\frac{d\phi}{dt} = \underbrace{\frac{n^2}{R_m + R_g}}_{L}\frac{di}{dt}$$

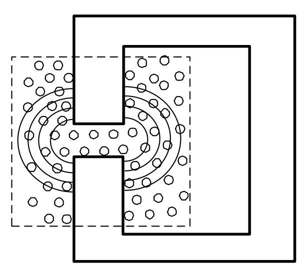
Current crowding

$$R_m = \frac{\ell_e}{\mu_r A_e}$$

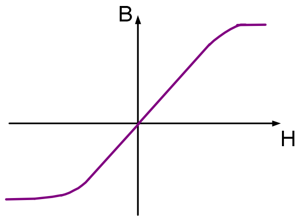
$$R_g = \frac{\ell_g}{\mu_0 A_e}$$

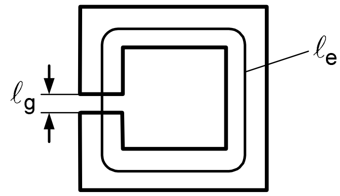
$$\frac{nI\ R_g}{\left(R_m + R_g\right)} \xrightarrow[R_g \gg R_m]{} nI$$





Inductor design
Air gap





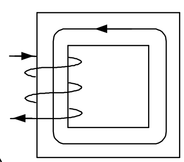
$$\ell_g \ll \ell_e$$

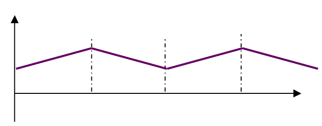
$$\ell_g + \ell_e \cong \ell_e$$

If $\dfrac{\ell_e}{\ell_g} < \mu_r$ $\quad \boxed{\mu_{re} \approx \left(\dfrac{\ell_e}{\ell_g}\right)}$

Inductor design
Saturation - Ae

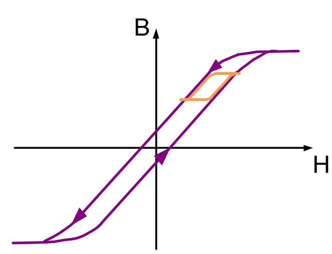
$$V = n\frac{d\Phi}{dt} \qquad V = L\frac{dI}{dt}$$





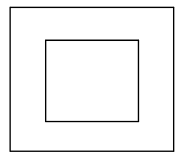
$$nA_e\int_0^{B_{\max}}\left(\frac{dB}{dt}\right)dt = L\int_0^{I_{pk}}\left(\frac{dI}{dt}\right)dt$$

$$LI_{pk} = nA_eB_{\max}$$



$$\boxed{n = \frac{\text{LIpk}}{\text{Bmax Ae}}}$$

Inductor design
Aw



$$A_w = \frac{nW_a}{k}$$

$$W_a = \frac{I_{rms}}{J}$$

$$A_w = \frac{nI_{rms}}{Jk}$$

Inductor design
Ap

$$A_p = A_e A_w = \frac{LI_{pk}I_{rms}}{B_{max}Jk}$$

Inductor design summary

- Calculate Ap
- Choose core
- Calculate n
- Calculate lg or adjust gap

Inductor selection Off-shelf product

VISHAY

IHLP-1616BZ-11

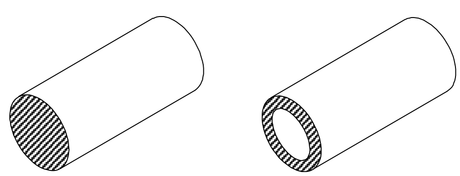
Vishay Dale

Low Profile,

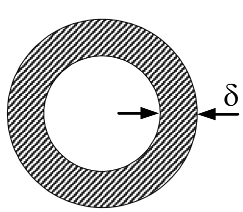
Inductor selection
Off-shelf product

STANDARD ELECTRICAL SPECIFICATIONS				
L_0 INDUCTANCE ± 20 % AT 100 kHz, 0.25 V, 0 A (μH)	DCR TYP. 25 °C (mΩ)	DCR MAX. 25 °C (mΩ)	HEAT RATING CURRENT DC TYP. (A) (3)	SATURATION CURRENT DC TYP. (A) (4)
0.10	4.1	4.5	12.0	12.0
0.22	6.5	7.0	9.0	9.0
0.47	14.5	16	7.0	7.0
1.0	24	27	4.5	5.0
2.2	61	68	3.25	3.25
4.7	95	105	1.7	1.75

Skin effect



DC High Frequency



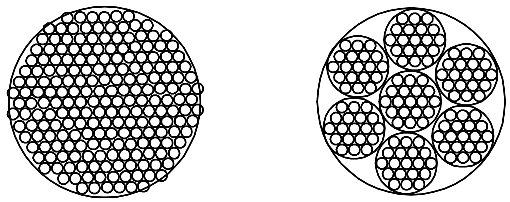
$$\frac{R_{AC}}{R_{DC}} > 1$$

δ – skin depth

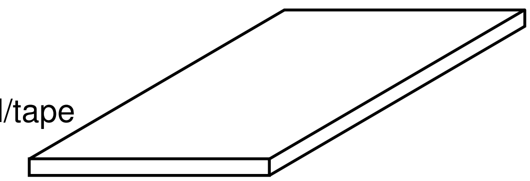
$$\delta\ (\text{mm}) = \frac{72}{\sqrt{f}}$$

f in Hz

Skin Effect Solutions

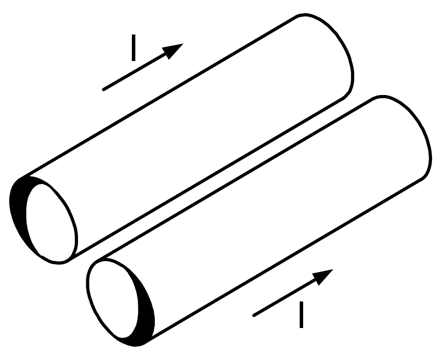


Litz wire



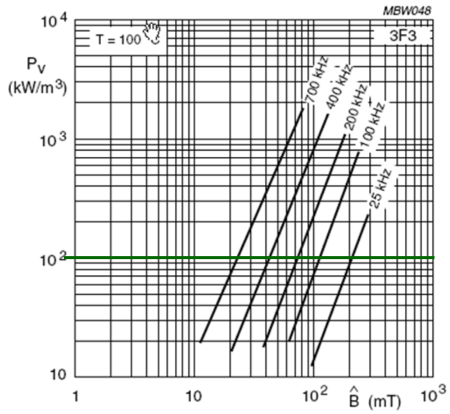
Foil/tape

Proximity effect



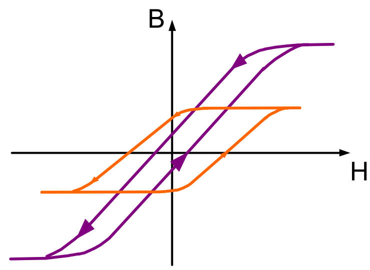
Current crowding due to magnetic fields

Losses function of ΔB

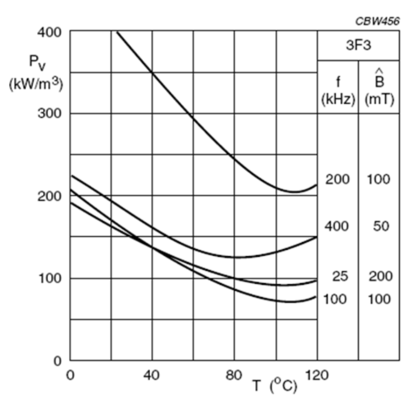


$P \sim K\, \Delta B^{\,i} f^{\,j}$

i, j – material dependent



Losses function of temperature



Losses
material selection

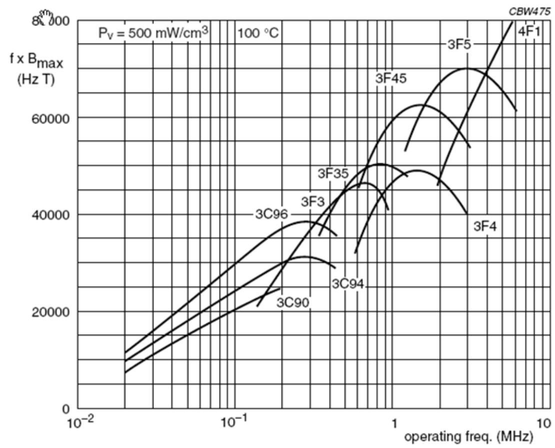
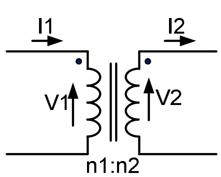


Fig.19 Performance factor ($f \times B_{max}$) at
PV = 500 mW/cm³ as a function of frequency for power ferrite materials.

Ideal transformer

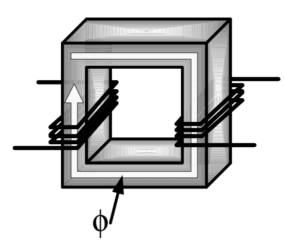


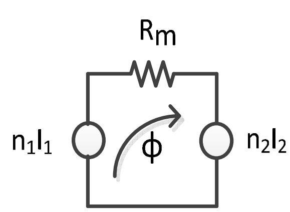
$$\phi R_m = n_1 I_1 - n_2 I_2$$

$$R_m \to 0$$

$$n_1 I_1 = n_2 I_2$$

$$V_1 = n_1 \frac{d\phi_1}{dt} \qquad V_2 = n_2 \frac{d\phi_2}{dt}$$

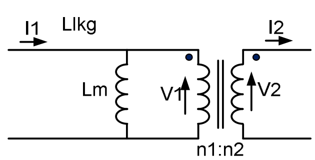


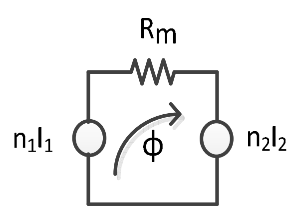


Transformer characteristics

- Current flow at the same time at primary and secondary
- Each winding represents inductance – Average voltage = 0
- AC only on any winding

Magnetizing inductance





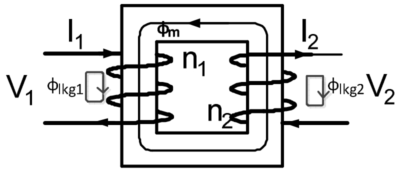
$$\phi R_m = n_1 I_1 - n_2 I_2$$

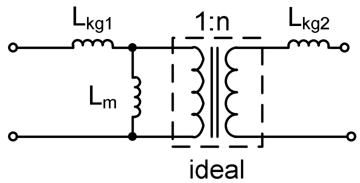
$$V_1 = n_1 \frac{d\phi_1}{dt}$$

$$V_1 = \underbrace{\frac{n_1^2}{R_m}}_{L_m} \frac{d}{dt}\left(i_1 + i_2 \frac{n_2}{n_1} \right)$$

Leakage inductance

- uncoupled magnetic flux





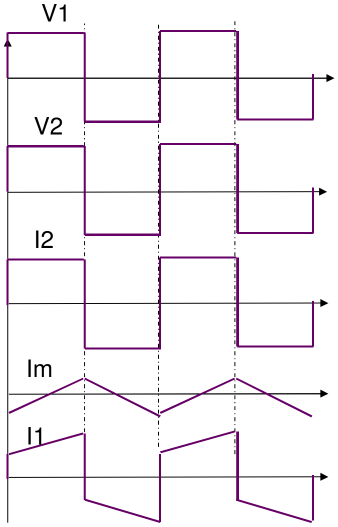
L_{lkg}, M (mutual coupling) and k (coupling coefficient)

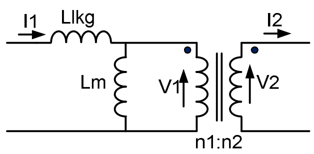
$$M = k\sqrt{L_1 \cdot L_2}$$

$$L_{lkg1} \cong L_m(1-k)$$

$$L_{lkg2} \cong L_{lkg1} \cdot n^2$$

Magnetizing and leakage inductances





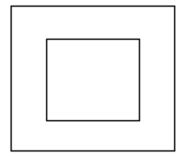
Transformer design
Aw

$$A_w = \frac{n_1 W_{a1} + n_2 W_{a2}}{k}$$

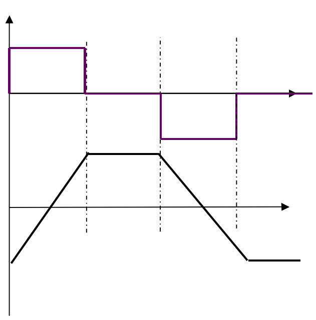
$$W_a = \frac{I_{rms}}{J}$$

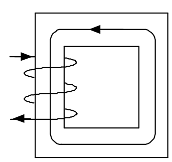
$$A_w = \frac{n_1 I_{1rms} + n_2 I_{2rms}}{Jk}$$

$$A_w = \frac{2 n_1 I_{1rms}}{Jk}$$



Transformer design
Saturation - Ae





$$B = \frac{1}{nAe} \int V dt$$

$$\left|B_{\max}{}^{+}\right| = \left|B_{\max}{}^{-}\right|$$

$$n = \frac{V_m T_{on}}{2 B_{\max} A_e}$$

Transformer design
Ap

$$A_p = A_w A_e = \frac{\{V_1 \cdot T_{on}\} 2 \cdot I_{1rms}}{\{2B_{\max}\} Jk}$$

$$A_p = \frac{\{V_1 \cdot T_{on}\} 2 \cdot I_{1_{rms}}}{\Delta B \cdot Jk}$$

$$A_p = \frac{\{V_1 \cdot D_{on}\} 2 \cdot I_{1_{rms}}}{f_s \cdot \Delta B \cdot Jk}$$

Transformer design
summary

- Calculate Ap
- Choose core
- Calculate n1
- Calculate n2

Magnetic design
Example

- Boost converter: P=100W, Vin=10V, Vo=50V, $\Delta IL=0.1I_{Lav}$
- Calculate L
- Calculate Ap