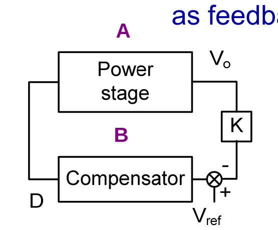
Control of switch-mode converters

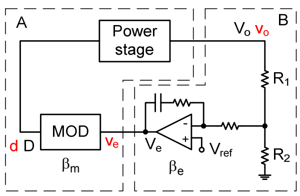
Control objectives

Produce control command to

- Regulate the output voltage
- Obtain zero or small steady-state (DC) error
- Quick response to reference changes
- Fast recovery
- Immunity to input and load changes
- Reasonable overshoot

Switch-mode converters as feedback systems

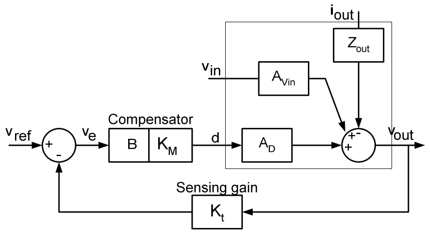




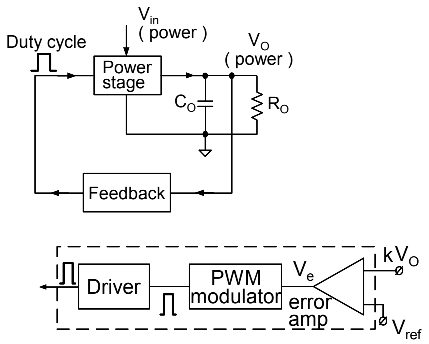
$\frac{v_o}{d}(f)$ Power stage $\frac{v_e}{v_o}(f)$ Compensator $\frac{d}{V_e}(f)$ Modulator

- Power stage is a Switching System (non-linear)
- Compensator is an analog or digital controller
- Linear control theory based design → small signal response

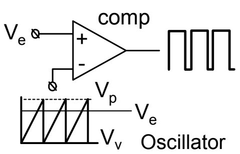
Control of PWM converters disturbances in voltage mode



Voltage regulation



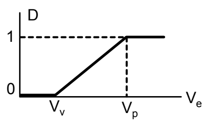
PWM modulator



$$V_t = \frac{(V_p - V_v)t}{T_s} + V_v$$

$$V_t = V_e = \frac{(V_p - V_v)t_{on}}{T_s} + V_v$$

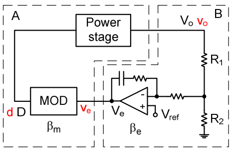
$$\frac{t_{on}}{T_s} = D_{on} = \frac{(V_e - V_v)}{V_p - V_v}$$

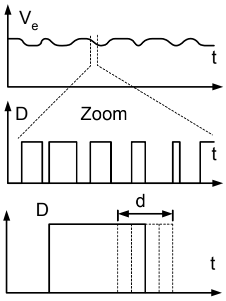


Practical $D_{on\ max} \approx 0.8 \div 0.9$

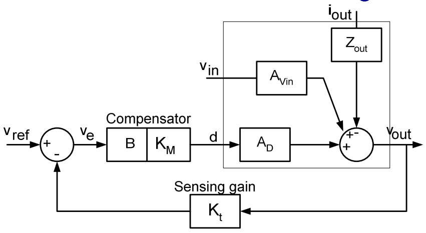
Sawtooth generator

Transfer functions





Control of PWM converters disturbances in voltage mode



$$A_d = \left.\frac{v_{out}}{d}\right|_{\substack{v_{in}=0 \\ i_{out}=0}}$$

$$A_{vin} = \left.\frac{v_{out}}{v_{in}}\right|_{\substack{d=0 \\ i_{out}=0}}$$

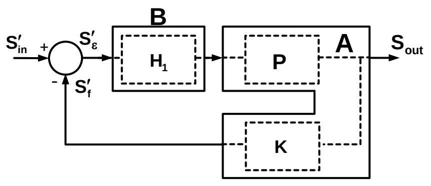
$$Z_{out} = \left.\frac{v_{out}}{i_{out}}\right|_{\substack{v_{in}=0 \\ d=0}}$$

$$v_{out} = dA_d + v_{in}A_{vin} - i_{out}Z_{out}$$

$$v_{out} = v_{ref}\frac{1}{K_t}\frac{LG}{1+LG} + v_{in}\frac{A_{vin}}{1+LG} - i_{out}\frac{Z_{out}}{1+LG}$$

$$LG = K_t K_M B A_d$$

Dynamics of feedback systems
Block diagram division



$$LG(f) = AB$$

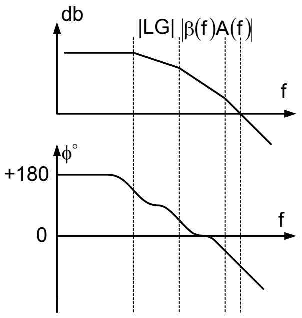
A – known (power stage + divider)
B – unknown (have to be designed)

LoopGain test
Nyquist Criterion

$$A_{CL} = \frac{A(s)}{1+LG(s)}$$

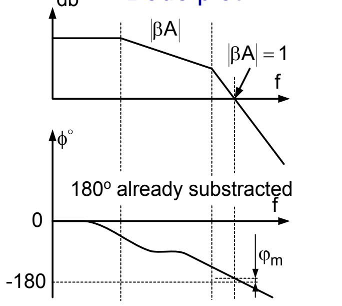
- The system is unstable if {1+LG(s)} has roots in the right half of the complex plane.
- Nyquist criterion is a test for location of {1+LG(s)} roots.
- Nyquist criterion is normally translated into the Bode plane (frequency domain)

LoopGain test



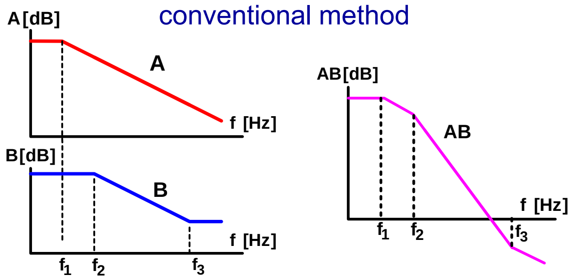
In negative feedback systems $\phi = 180^o\ (-180^o)$
At $f \rightarrow 0$

Bode plot



$$\varphi_m = \varphi_{|\beta A|=1} - (-180^\circ) = \varphi_{|\beta A|=1} + 180^\circ$$

Graphical representation of BA conventional method

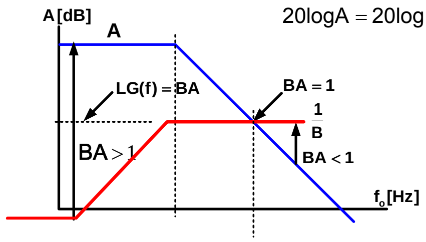


- Tedious – need to re-plot BA
- Analysis (not design) oriented
- Requires iterations

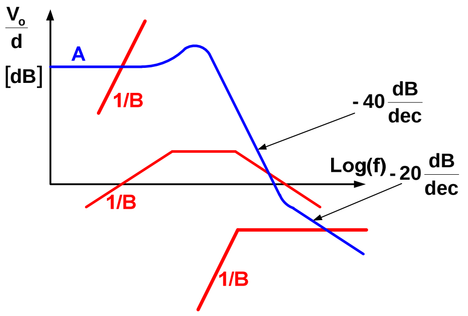
Graphical Representation of BA

$$20\log A - 20\log\frac{1}{B} = 20\log(BA)$$

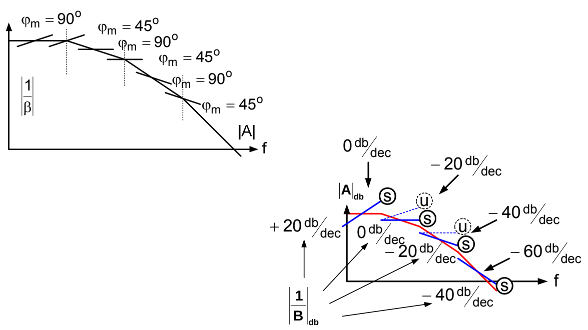
$$20\log A = 20\log\frac{1}{B} \Rightarrow B\cdot A = 1$$



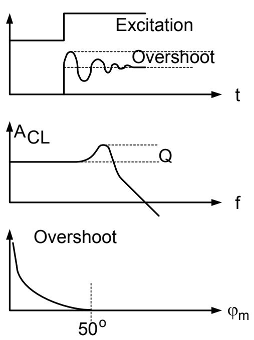
Possible compensations



Possible compensations



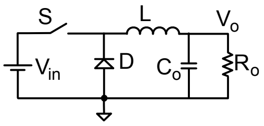
Overshoot and Q in Closed Loop in Response to step in S_{in}



$$Q \cong \frac{\sqrt{\cos\varphi_m}}{\sin\varphi_m} \quad \textbf{for} \ \varphi_m < 50^o$$

Design target $\varphi_m \geq 45^o$

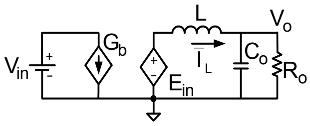
Extracting the power stage control-to-output transfer function



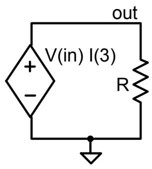
$E_{in} = V_{in} \cdot D_{on}$

$G_b = \overline{I}_L \cdot D_{on}$

$E_{in} - V_o \rightarrow \overline{V}_L$



Linearization

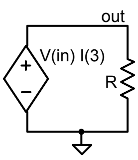


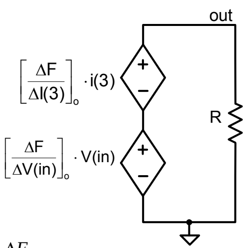
$V(out) = V(in) * I(3)$

$$d(V(out)) = \frac{\partial(V(out))}{\partial(V(in))} v(in) + \frac{\partial(V(out))}{\partial(I(3))} i(3)$$

$$V(out) = \frac{\Delta V(out)}{\Delta V(in)} v(in) + \frac{\Delta V(out)}{\Delta I(3)} i(3)$$

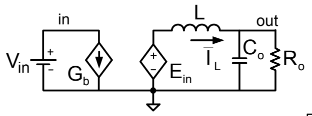
SPICE Linearization (AC Analysis)





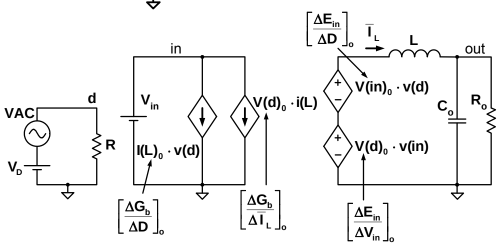
$\frac{\Delta F}{\Delta V(in)} = I(3)_0$ $\frac{\Delta F}{\Delta I(3)} = V(in)_0$

Buck linearization

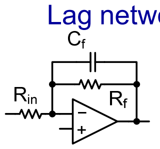


$E_{in} = V_{in}D$

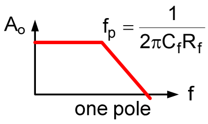
$G_b = \bar{I}_L D$



Possible phase compensation schemes
Lag network

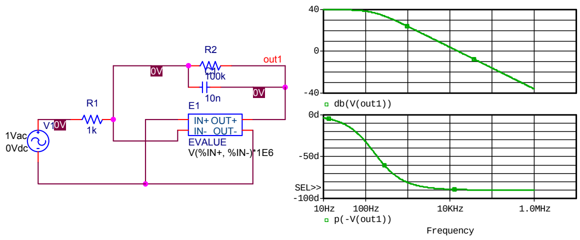


$$A_o = \frac{R_f}{R_{in}}$$

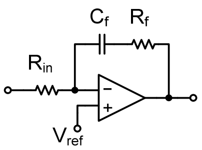


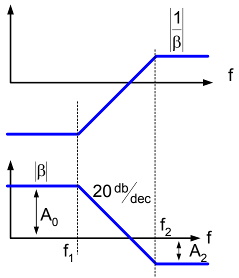
Design example

Lag network



Lag – Lead network



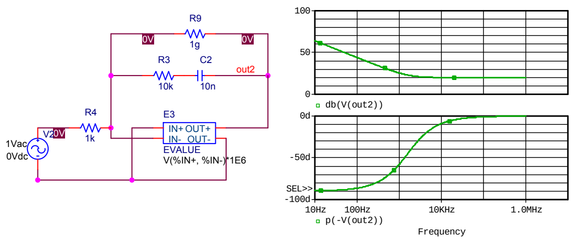


$$A_o = A_{OL}(ampl.)$$

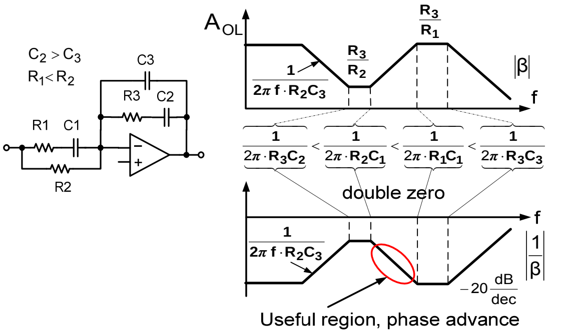
$$f_L = \frac{1}{2\pi C_f R_f}$$

$$A_2 = \frac{R_f}{R_{in}}$$

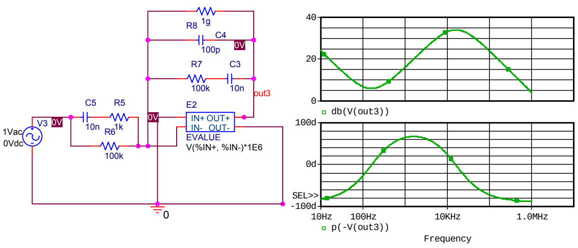
Lag-Lead network



Double zero compensation scheme

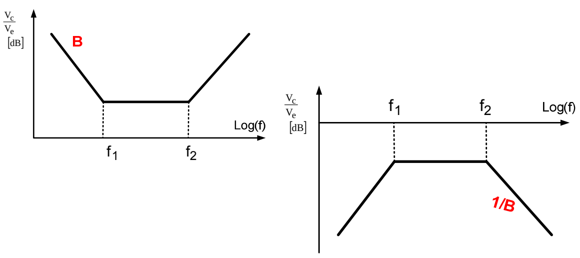


Double Zero



The relationship to PID compensators

$$\frac{v_c}{v_e} = K_p + \frac{K_I}{s} + s \cdot K_d = \frac{K_d}{K_I} \frac{(s+\omega_{z1}) \cdot (s+\omega_{z2})}{s}$$



The relationship to PID compensators